Examen de Econometria

Universidad de Montevideo

10 de Diciembre

Conteste solo 3 Preguntas

1. Let $\{x_t\}$ be an autoregressive process generated by the equation

$$x_t = x_{t-1} + \phi x_{t-2} + u_t,$$

where $\{u_t\}$ is a sequence of independent and identically distributed random variables with mean zero and variance $\sigma^2 < \infty$.

   (a) Explain what is meant by the Wold Decomposition Theorem. Under what conditions on ϕ does $\{x_t\}$ obey this theorem?

   (b) Derive the mean, variance and autocorrelation function of $\{x_t\}$. How do these results change when $\phi = 0$ (assume, if you wish, that the process starts at $t = 0$ with $x_0 = 0$)? Explain.

   (c) Derive the variance ratio statistic for the following moving average process

$$y_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + u_t,$$

   where $\{u_t\}$ is a white noise.

2. Suppose that the time series $\{x_t\}$ and $\{y_t\}$ are generated by the system

$$\begin{aligned} x_t + \beta y_t &= \varepsilon_{1t}, \qquad \varepsilon_{1t} = \varepsilon_{1,t-1} + u_{1t}, \\ x_t + \alpha y_t &= \varepsilon_{2t}, \qquad \varepsilon_{2t} = \rho\varepsilon_{2,t-1} + \rho\varepsilon_{2,t-2} + u_{2t}, \end{aligned}$$

where $\{(u_{1t}, u_{2t})'\}$ is a sequence of independent and identically distributed random vectors with zero mean and positive definite variance-covariance matrix

   (a) Under what conditions on ρ is ε_{2t} stationary?

   (b) Assuming that ε_{2t} is stationary, establish the orders of integration of $\{x_t\}$ and $\{y_t\}$. Are the two series cointegrated, and, if so, what is the cointegrating vector?

   (c) Derive the error-correction and the moving average representation of $(y_t, x_t)'$.

   (d) Outline procedures for testing for cointegration and discuss their advantages and disadvantages.

3. (a) Consider the following model:

$$
\begin{aligned}
y_t &= \mu + \varepsilon_t, \\
\varepsilon_t &= v_t(\alpha_0 + \alpha_1 \varepsilon_{t-1}^2)^{1/2}, \quad \text{where } v_t \sim N(0,1).
\end{aligned}
$$

Derive the first, second, third and fourth unconditional moments of ε_t.

(b) Consider the model

$$
\begin{aligned}
y_t &= \mu + \beta \varepsilon_{t-2} + \varepsilon_t, \\
\varepsilon_t &= v_t(\alpha_0 + \alpha_2 \varepsilon_{t-2}^2)^{1/2}, \quad \text{where } v_t \sim N(0,1).
\end{aligned}
$$

(i) Derive the conditional variance $\text{Var}(\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots)$.

(ii) Derive the unconditional variances $\text{Var}(\varepsilon_t)$ and $\text{Var}(y_t)$.

(iii) Under what assumption does $\text{Var}(y_t)$ exist?

(c) Explain how would you test the conditional CAPM, using a Multivariate GARCH model?

4. Consider the following VAR model

$$
\begin{bmatrix} y_t' \\ y_{t-1}' \\ x_t' \\ x_{t-1}' \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} \\ 1 & 0 & 0 & 0 \\ b_{11} & b_{12} & 0 & b_{14} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}' \\ y_{t-2}' \\ x_{t-1}' \\ x_{t-2}' \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \zeta_t \\ 0 \end{bmatrix},
$$

where ε_t and ζ_t are white noise processes, and y_t' and x_t' are given by

$$
\begin{aligned}
y_t' &= y_t - \alpha_0 - \alpha_1 S_t, \\
x_t' &= x_t - \alpha_0 - \alpha_2 S_t,
\end{aligned}
$$

and where S_t follows a two-state Markov process with transition probabilities

$$
\begin{aligned}
p &= P(S_t = 1 | S_{t-1} = 1), \\
q &= P(S_t = 0 | S_{t-1} = 0).
\end{aligned}
$$

(a) Derive the expected value of the state n periods ahead conditional on the information about the state at time t, $E(S_{t+n}|S_t)$.

(b) Suppose y_t is the first difference of the one-month interest rate (r_t), $y_t = r_t - r_{t-1}$, and x_t is the spread between the two-months interest rate (R_t) and the one-month interest rate, $x_t = R_t - r_t$. A version of the expectations hypothesis of the term structure of interest rates may be expressed as

$$
x_t = \frac{1}{2} E(y_{t+1}).
$$

Which testable restrictions does this theory impose on the parameters of the VAR and which on the switching parameters?

(c) Explain how to conduct specification tests for each of the equations of the VAR.