Mental Accounting for Multiple Outcomes: Theoretical Results and an Empirical Study

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Abstract. We develop a theoretical basis for guiding integration and segregation decisions in the case of multiple objects, or subjects, with underlying S-shaped value functions. Our findings show that Thaler’s principles of mental accounting work as postulated when the subjects are exposed to only positive experiences (e.g., gains) or negative experiences (e.g., losses). In the case of mixed-sign experiences, determining preferences for integration and segregation becomes a complex task, whose complete solution is provided in this paper in the case of three subjects, thus supplementing the only so far in the literature thoroughly developed case of two subjects. We illustrate our theoretical findings with examples, as well as with an experimental study, which we conducted at universities in Changchun, Hong Kong, and Paris.

Keywords: Integration, segregation, bundling, marketing, prospect theory, value function, decision making.

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1 Introduction

In a number of real-life situations, decision making involves combining and segregating multiple objects, units, or subjects, which we subsequently call ‘exposure units.’ Hence, given a value or utility function, our task is to determine if the exposure units should be combined or segregated, completely or partially, and in what combination.

The mental accounting framework of Thaler (1985) mainly examines consumer behavior in the case of two outcomes. Specifically, mental accounting is defined as “the set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities” (Thaler, 1999). The author defines a pattern of optimal behavior depending on the type of exposure units with positive or negative experiences, and provides an overall value of their combination or segregation in the case of two units.

Our goal in this paper is to facilitate a further understanding of Thaler’s principles, and especially of their validity in the case of more than two exposure units. To begin with the task, it is useful to recall a footnote in Thaler’s (1985, p. 202) ground-breaking work, where the author notes that “For simplicity I will deal only with two-outcome events, but the principles generalize to cases with several outcomes.” We can intuitively feel, however, that the expressed hope for a generalization may be too optimistic. The reason is that dealing with a large number of exposure units is a very complex and sophisticated cognitive process, much more complex than working with just two units, and thus we cannot remain certain that the mental accounting rules that worked well in the case of two exposure units would still work in the case of more than two units.

Hence, in this paper we investigate – both theoretically and empirically – the behavior of consumers who face more than two outcomes. Many real-life examples motivate this study. We may think of individuals who may need to choose between paying income taxes in one or several payments at different times, or we may think of a salary payment that allows one to choose between the benefit from a bonus by the employer each month and having income tax at the end of the fiscal year, or having one global bonus for the same amount together with income tax at the end of the fiscal year. All in all, in many situations we are required to decide whether segregation or integration of exposure units is preferable, and in what combination.

In general, suppose that we deal with $n \geq 2$ exposure units (e.g., events, items, products, consumers, etc.) whose experiences (values, ratings, happiness, etc.) are quantified and recorded in the form of real numbers $x_1, \ldots, x_n$, which can be negative and positive. For example, we may think of the experiences as losses or gains, measured in monetary units, or recorded on a Likert scale as is the case in our experimental study to be reported and
discussed later in this paper. The decision maker would like to know whether these \( n \) exposure units should be segregated or integrated, partially or totally, with the aim at having a combination of the exposure units with a most favourable total experience (e.g., maximal value).

The actual or perceived value of the experiences is represented by a value function, \( v \), which acts from the real line to the real line, is continuous and increasing, convex for non-positive experiences \((x \leq 0)\), and concave for non-negative experiences \((x \geq 0)\). In other words, our analysis is within the framework of prospect theory (Kahneman and Tversky, 1979). If, for example, two exposure units with experiences \( x_m \) and \( x_n \) are integrated, then their value is \( v(x_m + x_n) \), but if they are kept segregated, then the value is \( v(x_m) + v(x_n) \).

Thaler (1985) postulates four basic principles, also known as hedonic editing hypotheses, for integration and segregation:

P1. Segregate (two) exposure units with positive experiences.

P2. Integrate (two) exposure units with negative experiences.

P3. Integrate an exposure unit carrying a smaller negative experience with that carrying a larger positive experience.

P4. Segregate an exposure unit carrying a larger negative experience from that carrying a smaller positive experience.

When there are only two exposure units, then there can only be two possibilities: either integrate both or segregate both. For a detailed analysis of this case within the prospect theory, we refer to Egozcue and Wong (2010). These authors find, for example, that when facing small positive experiences and large negative ones, loss averters sometimes prefer to segregate, sometimes to integrate, and at other times stay neutral. For a detailed theoretical and experimental analysis of principle P4, which is also known as the “silver lining effect,” we refer to Jarnebrant et al. (2009).

There are some empirical works that study decision maker’s behavior in the case of multiple exposure units. For example, Loughran and Ritter (2002), Ljungqvist and Wilhelm (2005) examine how mental accounting of multiple outcomes affects the behavior of market participants in various contexts of finance. However, as far as we know, there has not been a theory that would completely sort out the behavior of investors for mental accounting in the case of multiple exposure units. The present paper aims at developing such results within the prospect theory.

We have organized the paper as follows. In Section 2 we present basic definitions as well as a theorem that solves the integration/segregation problem when all experiences are of
same sign. In the same section we also consider the situation when experiences may have varying signs but only two choices are available to the value maximizing decision maker: either integrate all or segregate all. (In the case of only two exposure units, this provides a description of optimal decisions concerning integration and segregation.) In Section 3 we give a thorough solution of the integration and segregation problem in the case of three exposure units. In Section 4, we offer numerical examples illustrating our theoretical results and showing their optimality; this section also contains details of an experiment that we have conducted at universities in Changchun, Hong Kong, and Paris. Section 5 offers concluding remarks. An appendix contains further details related to the aforementioned experiment.

2 Basics

2.1 Value functions

As we have noted in the introduction, we work within the prospect theory and thus deal with value functions $v$ that are continuous and increasing, convex on $(-\infty, 0]$ and concave on $[0, \infty)$. Specifically, let

$$v(x) = \begin{cases} v_+(x) & \text{when } x \geq 0, \\ -v_-(x) & \text{when } x < 0, \end{cases}$$

(2.1)

where $v_-, v_+ : [0, \infty) \to [0, \infty)$ are two increasing and concave functions such that $v_-(0) = 0 = v_+(0)$, $v_-(x) > 0$ and $v_+(x) > 0$ for all $x > 0$. For example, Kahneman and Tversky (1979) use a value function that is equal to $x^{\gamma_G}$ when $x \geq 0$ and $-\lambda(-x)^{\gamma_L}$ when $x < 0$, where $\lambda > 0$ is the degree of loss aversion, and $\gamma_G$ and $\gamma_L \in (0, 1)$ are degrees of diminishing sensitivity. Assuming preference for homogeneity and loss aversion, Al-Nowaihi et al. (2008) demonstrate that $\gamma_G$ and $\gamma_L$ are equal; denote them by $\gamma \in (0, 1)$. This naturally leads us to the following class of value functions:

$$v_\lambda(x) = \begin{cases} u(x) & \text{when } x \geq 0, \\ -\lambda u(-x) & \text{when } x < 0, \end{cases}$$

(2.2)

where $u : [0, \infty) \to [0, \infty)$ is continuous, concave, and such that $u(0) = 0$ and $u(x) > 0$ for all $x > 0$, with $\lambda > 0$ reflecting loss aversion. We shall work with the latter value function throughout this paper.

2.2 Petrović’s inequality

From the mathematical point of view, integration and segregation are about superadditivity and subadditivity of the value function. However, decision makers ‘visualize’ value functions
in terms of their shapes: e.g., concave or convex in one region or another. A link between additivity and concavity type notions is provided by a theory of functional inequalities, among which we have Petrović’s inequality (see, e.g., Kuczma, 2009)

\[ v\left(\sum_{k=1}^{n} x_k\right) \leq \sum_{k=1}^{n} v(x_k), \]  

(2.3)

that holds for all continuous and concave functions \( v : [0, \infty) \rightarrow \mathbb{R} \) such that \( v(0) = 0 \), and for all \( n \geq 2 \) and \( x_1, \ldots, x_n \in [0, \infty) \). In other words, inequality (2.3) means that the function \( v \) is subadditive on \([0, \infty), which implies that the value maximizing decision maker prefers to segregate positive experiences. In the domain \((-\infty, 0]\) of losses, the roles of integration and segregation are reversed. In the case of the value function \( v_\lambda(x) \), this follows from the easily checked equation

\[ v_\lambda(x) = -\lambda v_{1/\lambda}(-x), \]  

(2.4)

which holds for all real \( x \). We shall find this ‘reflection’ equation particularly useful later in this paper. Now, for a later convenient referencing, we collect the above observations concerning total integration and total segregation into a theorem.

**Theorem 2.1** The value maximizing decision maker with any value function \( v \) given by equation (2.1) prefers to segregate any finite number of exposure units with positive experiences, and integrate any finite number of exposure units with negative experiences.

In the case of at least one positive and at least one negative experiences, deciding whether to integrate or segregate becomes a complex task, well beyond the simplicity of Theorem 2.1. Indeed, as demonstrated by Egozcue and Wong (2010) in the case \( n = 2 \), the value maximizing decision maker may prefer integrating some mixed experiences and segregate others, depending on how large or small they are. In the following sections we shall provide further results on the topic.

### 2.3 A threshold

In this subsection we investigate the problem of whether it is better to integrate all \( n \geq 2 \) exposure units or keep them segregated, assuming that – for whatever reason – these are the only two options available to the decision maker. From now on throughout this paper, we work with the value function \( v_\lambda(x) \).

The following quantity, denoted by \( T(x) \) and called threshold, plays a pivotal role in our
considerations:

\[
T(\mathbf{x}) = \frac{\sum_{k \in \mathcal{K}_+} u(x_k) - u\left(\max\left\{0, \sum_{k=1}^{n} x_k\right\}\right)}{\sum_{k \in \mathcal{K}_-} u(-x_k) - u\left(\max\left\{0, -\sum_{k=1}^{n} x_k\right\}\right)},
\]

where \(\mathcal{K}_+ = \{k : x_k > 0\}\) and \(\mathcal{K}_- = \{k : x_k < 0\}\) are subsets of \(\{1, \ldots, n\}\), and \(\mathbf{x} = (x_1, \ldots, x_n)\) is the vector of exposures.

**Theorem 2.2** The threshold \(T(\mathbf{x})\) is always non-negative. It splits the set of \(\lambda > 0\) values into two regions: integration and segregation. Namely, assuming that there is at least one exposure unit with a positive experience and at least one with a negative experience, then, given that only two options – either integrate or segregate all exposure units – are available to the decision maker, the value maximizing decision maker prefers to:

- integrate all exposure units if and only if \(T(\mathbf{x}) \leq \lambda\);
- segregate all exposure units if and only if \(T(\mathbf{x}) \geq \lambda\).

Consequently, the decision maker is indifferent to integration and segregation whenever \(T(\mathbf{x}) = \lambda\).

**Proof.** We start with the case \(\sum_{k=1}^{n} x_k \geq 0\). The inequality \(v_\lambda(\sum_{k=1}^{n} x_k) \leq \sum_{k=1}^{n} v_\lambda(x_k)\) is equivalent to

\[
u\left(\sum_{k=1}^{n} x_k\right) \leq -\lambda \sum_{k \in \mathcal{K}_-} u(-x_k) + \sum_{k \in \mathcal{K}_+} u(x_k),
\]

which is equivalent to

\[
\lambda \leq T_+(\mathbf{x}) \equiv \frac{\sum_{k \in \mathcal{K}_+} u(x_k) - u\left(\sum_{k=1}^{n} x_k\right)}{\sum_{k \in \mathcal{K}_-} u(-x_k)}.
\]

(2.5)

Since \(\sum_{k=1}^{n} x_k \geq 0\), we have that \(T_+(\mathbf{x}) = T(\mathbf{x})\). We shall next show that \(T(\mathbf{x})\) is non-negative. This is equivalent to showing that the numerator on the right-hand side of bound (2.5) is non-negative. For this, we first note that since the function \(u\) is non-decreasing and \(\sum_{k \in \mathcal{K}_-} x_k \leq 0\), we have that

\[
\sum_{k \in \mathcal{K}_+} u(x_k) - u\left(\sum_{k=1}^{n} x_k\right) = \sum_{k \in \mathcal{K}_+} u(x_k) - u\left(\sum_{k \in \mathcal{K}_+} x_k + \sum_{k \in \mathcal{K}_-} x_k\right) \\
\geq \sum_{k \in \mathcal{K}_+} u(x_k) - u\left(\sum_{k \in \mathcal{K}_+} x_k\right).
\]

(2.6)
Since the function \( u : [0, \infty) \to \mathbb{R} \) is continuous, concave, and \( u(0) = 0 \), the right-hand side of bound (2.6) is non-negative. Hence, \( T_+(x) \geq 0 \).

Consider now the case \( \sum_{k=1}^{n} x_k \leq 0 \). Then \( v_\lambda(\sum_{k=1}^{n} x_k) \leq \sum_{k=1}^{n} v_\lambda(x_k) \) is equivalent to

\[
\lambda \sum_{k \in K_-} u(-x_k) - \lambda u \left( -\sum_{k=1}^{n} x_k \right) \leq \sum_{k \in K_+} u(x_k). \tag{2.7}
\]

To rewrite inequality (2.7) with only \( \lambda \) on the left-hand side, we prove that the quantity

\[
\sum_{k \in K_-} u(-x_k) - \sum_{k \in K_-} u \left( -\sum_{k=1}^{n} x_k \right)
\]

is non-negative. Since the function \( u \) is non-decreasing and \( \sum_{k=1}^{n} x_k \geq 0 \), we have that

\[
\sum_{k \in K_-} u(-x_k) - \sum_{k \in K_-} u \left( -\sum_{k=1}^{n} x_k \right) \geq \sum_{k \in K_-} u(-x_k) - \sum_{k \in K_-} u \left( -\sum_{k=1}^{n} x_k \right). \tag{2.8}
\]

Since the function \( u : [0, \infty) \to \mathbb{R} \) is continuous, concave, and \( u(0) = 0 \), the right-hand side of bound (2.8) is non-negative. Hence, inequality (2.7) is equivalent to

\[
\lambda \leq T_-(x) \equiv \sum_{k \in K_+} u(x_k) \frac{\sum_{k \in K_-} u(-x_k) - \sum_{k \in K_-} u \left( -\sum_{k=1}^{n} x_k \right)}{\sum_{k \in K_-} u(-x_k) - \sum_{k=1}^{n} x_k}. \tag{2.9}
\]

Given the above, we have that \( T_-(x) \geq 0 \). Furthermore, since \( \sum_{k=1}^{n} x_k \leq 0 \), we have that \( T_-(x) = T(x) \). This completes the proof of Theorem 2.2. \( \square \)

Since there can only be either complete integration or complete segregation when \( n = 2 \), the threshold \( T(x) \) plays a decisive role in determining optimal strategies in the case of only two exposure units. For this reason, and also for having a convenient reference, we now reformulate Theorem 2.2 in the case \( n = 2 \). Namely, let \( x = (x_+, x_-) \) with \( x_+ > 0 \) and \( x_- < 0 \). Using the notation \( T(x_+, x_-) \) instead of \( T(x) \), we have the following expression for the threshold:

\[
T(x_+, x_-) = \frac{u(x_+) - u \left( \max \left\{ 0, x_- + x_+ \right\} \right)}{u(-x_-) - u \left( \max \left\{ 0, -(x_- + x_+) \right\} \right)}. \tag{2.10}
\]

Hence, having only the choices of either \( v_\lambda(x_- + x_+) \) or \( v_\lambda(x_-) + v_\lambda(x_+) \), the value maximizing decision maker prefers \( v_\lambda(x_- + x_+) \) if and only if \( T(x_+, x_-) \leq \lambda \), and prefers \( v_\lambda(x_-) + v_\lambda(x_+) \) if and only if \( T(x_+, x_-) \geq \lambda \).

Next we provide an additional insight into the case \( n = 2 \) with a result on the magnitude of \( T(x) \).
**Theorem 2.3** Let the conditions of Theorem 2.2 be satisfied, and let \( n = 2 \). (Thus, in particular, one among \( x_1 \) and \( x_2 \) is positive and another is negative.) If \( x_1 + x_2 \geq 0 \), then \( T(x) \leq 1 \), and if \( x_1 + x_2 \leq 0 \), then \( T(x) \geq 1 \).

**Proof.** We start with the case \( x_1 + x_2 \geq 0 \). Then \( T(x) \) is equal to \( T_+(x) \), which is defined on the right-hand side of inequality (2.5). Hence, the bound \( T(x) \leq 1 \) is equivalent to

\[
u(x_+ - x_-) \leq u(-x_-).
\] (2.11)

With the notation \( y_1 = x_- \geq 0 \) and \( y_2 = x_- + x_+ \geq 0 \), we rewrite inequality (2.11) as

\[
u(y_1 + y_2) \leq u(y_1) + u(y_2).
\] (2.12)

By Theorem 2.1, inequality (2.12) holds because \( u : [0, \infty) \to \mathbb{R} \) is continuous, concave, and \( u(0) = 0 \). This proves that \( T(x) \leq 1 \).

Consider now the case \( x_1 + x_2 \leq 0 \). Then \( T(x) \) is equal to \( T_-(x) \), which is defined on the right-hand side of inequality (2.9). Hence, the bound \( T(x) \geq 1 \) is equivalent to

\[
u(x_+ - x_-) \geq u(-x_-).
\] (2.13)

With the notation \( z_1 = x_+ \geq 0 \) and \( z_2 = -x_- - x_+ \geq 0 \), inequality (2.13) becomes

\[
u(z_1 + z_2) \geq u(z_1 + z_2).
\] (2.14)

By Theorem 2.1, inequality (2.14) holds, and so we have \( T(x) \geq 1 \). This completes the proof of Theorem 2.3. \( \square \)

### 3 Case \( n = 3 \): which ones to integrate, if any?

Complete integration and complete segregation may not result in the maximal value, and thus partial integration or segregation may be desirable. We shall next present a complete solution to this problem in the case of three exposure units, that is, when \( n = 3 \). The solution will be complex, but we have presented it in such a way that it would be readily implementable in the computer.

Unless explicitly noted otherwise, throughout this section we denote the corresponding three experiences by \( x, y \) and \( z \), and assume that

\[
x + y + z \geq 0.
\] (3.1)

Furthermore, without loss of generality we assume that

\[
x \geq y \geq z,
\] (3.2)
since every other case can be reduced to (3.2) by simply changing the notation. Finally, we can and thus do assume that

$$x \neq 0, \ y \neq 0, \ z \neq 0,$$  

(3.3)
because if at least one of the three experiences is zero, then the currently investigated case \( n = 3 \) reduces to \( n = 2 \), which has been discussed earlier in this paper and also investigated by Egozcue and Wong (2010).

There are five possibilities for integration and segregation in the case of three exposure units:

(A) \( v_\lambda(x) + v_\lambda(y) + v_\lambda(z) \)

(B) \( v_\lambda(x) + v_\lambda(y + z) \)

(C) \( v_\lambda(y) + v_\lambda(x + z) \)

(D) \( v_\lambda(z) + v_\lambda(x + y) \)

(E) \( v_\lambda(x + y + z) \)

We want to determine which of these five possibilities, and under what conditions, produces the largest value. We also want to know which of them, and under what conditions, produces the smallest value. This makes the contents of Theorems 3.1–3.5 below.

In Theorems 3.1–3.5 and their proofs, we shall use notation such as \( (A) \succ (E) \), which means that \( v_\lambda(x) + v_\lambda(y) + v_\lambda(z) \geq v_\lambda(x + y + z) \). That is, \( (A) \succ (E) \) is a concise way of saying that the value maximizing decision maker prefers (A) to (E).

**Note 3.1** The reason for including minimal values when only the maximal ones seem to be of real interest is based on the fact that finding maximal ones in the case \( x + y + z \leq 0 \) can be reduced to finding minimal ones under the condition \( x + y + z \geq 0 \). Indeed, note that \( x + y + z \leq 0 \) is equivalent to \( x^- + y^- + z^- \geq 0 \) with the notation \( x^- = -x, \ y^- = -y, \) and \( z^- = -z \). Next, since \( \lambda > 0 \), equation (2.4) implies that finding the maximal value among (A)–(E) is equivalent to finding the minimal value among the following five ones:

\[
\begin{align*}
v_{1/\lambda}(x^-) + v_{1/\lambda}(y^-) + v_{1/\lambda}(z^-) \\
v_{1/\lambda}(x^-) + v_{1/\lambda}(y^- + z^-) \\
v_{1/\lambda}(y^-) + v_{1/\lambda}(x^- + z^-) \\
v_{1/\lambda}(z^-) + v_{1/\lambda}(x^- + y^-) \\
v_{1/\lambda}(x^- + y^- + z^-)
\end{align*}
\]
The minimal values among these five possibilities can easily be derived from Theorems 3.1–3.5 below, where we only need to replace $x$, $y$, and $z$ by $x^-$, $y^-$, and $z^-$, respectively, and also the parameter $\lambda$ by $1/\lambda$. □

Hence, throughout this section we are only concerned with the case $x + y + z \geq 0$, and thus have that at least one of the three exposure units has a non-negative experience. Furthermore, note that every triplet $x$, $y$, and $z$ falls into one of the following five cases:

- $x \geq y \geq z \geq 0$ (3.4)
- $x \geq y \geq 0 \geq z$ and $y \geq -z$ (3.5)
- $x \geq y \geq 0 \geq z$ and $x \geq -z \geq y$ (3.6)
- $x \geq y \geq 0 \geq z$ and $-z \geq x$ (3.7)
- $x \geq 0 \geq y \geq z$ (3.8)

**Theorem 3.1** Let the value function be $v_\lambda$, and let $x \geq y \geq z \geq 0$. Then we have the following two statements:

Max: (A) gives the maximal value among (A)–(E).

Min: (E) gives the minimal value among (A)–(E).

**Proof.** Since the three exposure units have non-negative experiences $x$, $y$, and $z$, Theorem 2.1 implies that complete segregation maximizes the value. Hence, (A) attains the maximal value among (A)–(E). Same theorem also implies that complete integration, which is (E), attains the minimal value. □

The following analysis of cases (3.5)–(3.8) is much more complex. We shall frequently use a special case of the Hardy-Littlewood-Pólya (HLP) majorization principle (e.g., Kuczma, 2009, p. 211). Namely, given two vectors $(x_1, x_2)$ and $(y_1, y_2)$, and also a continuous and concave function $v$, we have the implication:

$$\begin{align*}
x_1 \geq x_2, & \quad y_1 \geq y_2 \\
x_1 + x_2 = y_1 + y_2 \\
x_1 \leq y_1
\end{align*} \quad \Rightarrow \quad v(x_1) + v(x_2) \geq v(y_1) + v(y_2). \quad (3.9)$$

Now we are ready to formulate and prove our next theorem.

**Theorem 3.2** Let the value function be $v_\lambda$, and let $x \geq y \geq 0 \geq z$ with $y \geq -z$.

Max: With the threshold $T_{AC} = T(x, z)$, the following statements specify the two possible maximal values among (A)–(E):
- When $T_{AC} \geq \lambda$, then (A).
- When $T_{AC} \leq \lambda$, then (C).

Min: With the threshold $T_{DE} = T(x + y, z)$, the following statements specify the two possible minimal values among (A)–(E):

- When $T_{DE} \geq \lambda$, then (E).
- When $T_{DE} \leq \lambda$, then (D).

**Proof.** Since $x$ and $y$ are non-negative, from Theorem 2.1 we have that $(A) \succ (D)$, and since $x$ and $y + z$ are non-negative, the same theorem implies that $(B) \succ (E)$. The proof of $(C) \succ (B)$ is more complex. Note that $(C) \succ (B)$ is equivalent to

$$v_{\lambda}(y) + v_{\lambda}(x + z) \geq v_{\lambda}(x) + v_{\lambda}(y + z), \quad (3.10)$$

which we establish as follows:

- When $x + z \geq y$, then we apply the HLP principle on the vectors $(x + z, y)$ and $(x, y + z)$ and get $v_{\lambda}(x + z) + v_{\lambda}(y) \geq v_{\lambda}(x) + v_{\lambda}(y + z)$, which is (3.10).

- When $x + z \leq y$, then we apply the HLP principle on the vectors $(y, x + z)$ and $(x, y + z)$, and get $v_{\lambda}(y) + v_{\lambda}(x + z) \geq v_{\lambda}(x) + v_{\lambda}(y + z)$, which is (3.10).

This completes the proof of inequality (3.10). Hence, in order to establish the ‘max’ part of Theorem 3.2, we need to determine whether (A) or (C) is maximal, and for the ‘min’ part, we need to determine whether (D) or (E) is minimal.

**The ‘max’ part.** Since $x \geq 0$ and $z \leq 0$, whether (A) or (C) is maximal is determined by the threshold $T_{AC}$: when $T_{AC} \leq \lambda$, then $(C) \succ (A)$, and when $T_{AC} \geq \lambda$, then $(A) \succ (C)$.

This concludes the proof of the ‘max’ part.

**The ‘min’ part.** Since $x + y \geq 0$ and $z \leq 0$, the threshold $T_{DE} = T(x + y, z)$ plays a decisive role: if $T_{DE} \leq \lambda$, then $(E) \succ (D)$, and if $T_{DE} \geq \lambda$, then $(D) \succ (E)$. This concludes the proof of the ‘min’ part and of Theorem 3.2 as well. □

**Theorem 3.3** Let the value function be $v_{\lambda}$, and let $x \geq y \geq 0 \geq z$ with $x \geq -z \geq y$.

Max: With the threshold $T_{AC} = T(x, z)$, the following statements specify the two possible maximal values among (A)–(E):

- When $T_{AC} \geq \lambda$, then (A).
When $T_{AC} \leq \lambda$, then (C).

Min: With the thresholds $T_{BE} = T(x, y + z)$, $T_{DE} = T(x + y, z)$, and

$$T_{BD} = \frac{u(x + y) - u(x)}{u(-z) - u(-y - z)},$$

the following statements specify the three possible minimal values among (A)–(E):

- When $T_{BE} \leq \lambda$ and $T_{BD} \geq \lambda$, then (B).
- When $T_{DE} \leq \lambda$ and $T_{BD} \leq \lambda$, then (D).
- When $T_{BE} \geq \lambda$ and $T_{DE} \geq \lambda$, then (E).

Proof. Since $x$ and $y$ are non-negative, we have (A) $\succ$ (D), and since $y$ and $x + z$ are non-negative, we have (C) $\succ$ (E). Hence, only (A), (B), and (C) remain to consider for the ‘max’ part of the theorem, and (B), (D), and (E) for the ‘min’ part.

The ‘max’ part. First we show that $T_{AC} \leq T_{AB}$. Since $x + z \geq 0$, from Theorem 2.3 we have $T_{AC} \leq 1$, and since $y + z \leq 0$, the same theorem implies $T_{AB} \geq 1$. Hence, $T_{AC} \leq T_{AB}$.

To establish that (A) is maximal when $T_{AC} \geq \lambda$, we check that (A) $\succ$ (B) and (A) $\succ$ (C). The former statement holds when $T_{AB} = T(y, z) \geq \lambda$, and the latter when $T_{AC} = T(x, z) \geq \lambda$. But we already know that $T_{AC} \leq T_{AB}$. Therefore, when $T_{AC} \geq \lambda$, then $T_{AB} \geq \lambda$. This proves that when $T_{AC} \geq \lambda$, then (A) gives the maximal value among (A), (B), (C), and thus, in turn, among all (A)–(E).

To establish that (C) is the maximal when $T_{AC} \leq \lambda$, we need to check that (C) $\succ$ (A) and (C) $\succ$ (B). First we note that when $T_{AC} \leq \lambda$, then (C) $\succ$ (A). Furthermore,

$$v_{\lambda}(x) + v_{\lambda}(y + z) \leq v_{\lambda}(y) + v_{\lambda}(x + z) \iff u(x) - \lambda u(-y - z) \leq u(y) + u(x + z)$$

$$\iff T_{BC} \leq \lambda,$$

where $T_{BC}$ is defined by the equation

$$T_{BC} = \frac{u(x) - u(x + z) - u(y)}{u(-y - z)}.$$

Hence, when $T_{BC} \leq \lambda$, then (C) $\succ$ (B). Simple algebra shows that the bound $T_{BC} \leq T_{AB}$ is equivalent to $T_{AC} \leq T_{AB}$, and we already know that the latter holds. Hence, $T_{BC} \leq T_{AB}$ and so $T_{BC} \leq \lambda$ when $T_{AC} \leq \lambda$. In summary, when $T_{AC} \leq \lambda$, then (C) gives the maximal value among all cases (A)–(E). This concludes the proof of the ‘max’ part.
The ‘min’ part. We first establish conditions under which (B) is minimal. We have \((E) \succ (B)\) when \(T_{BE} \leq \lambda\). To have \((D) \succ (B)\), we need to employ the threshold \(T_{BD}\), which is defined in the formulation of the theorem. The role of the threshold is seen from the following equivalence relations:

\[
v_{\lambda}(x) + v_{\lambda}(y + z) \leq v_{\lambda}(z) + v_{\lambda}(x + y) \iff u(x) - \lambda u(-y - z) \leq -\lambda u(-z) + u(x + y) \iff \lambda \leq T_{BD}.
\]

Hence, if \(T_{BD} \geq \lambda\), then \((D) \succ (B)\). In summary, when \(T_{BE} \leq \lambda\) and \(T_{BD} \geq \lambda\), then (B) gives the minimal value among all (A)–(E).

We next establish conditions under which (D) is minimal. First, we have \((E) \succ (D)\) when \(T_{DE} \leq \lambda\). Next, we have \((B) \succ (D)\) when \(T_{BD} \leq \lambda\). In summary, when \(T_{DE} \leq \lambda\) and \(T_{BD} \leq \lambda\), then (D) gives the minimal value among all (A)–(E).

Finally, we have \((B) \succ (E)\) when \(T_{BE} \geq \lambda\), and \((D) \succ (E)\) when \(T_{DE} \geq \lambda\). Hence, when \(T_{BE} \geq \lambda\) and \(T_{DE} \geq \lambda\), then (E) is minimal among all (A)–(E). This finishes the proof of the ‘min’ part, and thus of Theorem 3.3 as well. \(\square\)

**Theorem 3.4** Let the value function be \(v_{\lambda}\), and let \(x \geq y \geq 0 \geq z\) with \(-z \geq x\).

Max: With the threshold

\[
T_{AE} = \frac{u(x) + u(y) - u(x + y + z)}{u(-z)},
\]

the following statements specify the two possible maximal values among (A)–(E):

- When \(T_{AE} \geq \lambda\), then (A).
- When \(T_{AE} \leq \lambda\), then (E).

Min: With the thresholds \(T_{AC} = T(x, z)\), \(T_{BE} = T(x, y + z)\), \(T_{CE} = T(y, x + z)\), \(T_{DE} = T(x + y, z)\), and

\[
T_{BC} = \frac{u(x) - u(y)}{u(-y - z) - u(-x - z)},
T_{BD} = \frac{u(x + y) - u(x)}{u(-z) - u(-y - z)},
T_{CD} = \frac{u(x + y) - u(y)}{u(-z) - u(-x - z)},
\]

the following statements specify the four possible minimal values among (A)–(E):

- When \(T_{BE} \leq \lambda\), \(T_{BC} \leq \lambda\), and \(T_{BD} \geq \lambda\), then (B).
- When \(T_{CE} \leq \lambda\), \(T_{BC} \geq \lambda\), and \(T_{CD} \geq \lambda\), then (C).
When $T_{DE} \leq \lambda$, $T_{BD} \leq \lambda$, and $T_{CD} \leq \lambda$, then (D).

When $T_{BE} \geq \lambda$, $T_{CE} \geq \lambda$, and $T_{DE} \geq \lambda$, then (E).

**Proof.** Since both $x$ and $y$ are non-negative, we have $(A) \succ (D)$. This eliminates (D) from the ‘max’ part of Theorem 3.4 and (A) from the ‘min’ part.

**The ‘max’ part.** We first eliminate (B). When $T_{BE} \leq \lambda$, then $(D) \succ (B)$. If, however, $T_{BE} \geq \lambda$, then by Theorem 2.2 we have $(B) \succ (E)$. We shall next show that in this case we also have $(A) \succ (B)$, thus making (B) unattractive to the value maximizing decision maker. Since $y + z \leq 0$ and $x + y + z \geq 0$, we have from Theorem 2.3 that $T_{BE} \leq 1$. Theorem 2.3 also implies that $T_{AC} \geq 1$ because $x + z \leq 0$. Hence, $T_{BE} \leq T_{AB}$. Since $T_{BE} \geq \lambda$, we conclude that $T_{AB} \geq \lambda$. By Theorem 2.2, the latter bound implies $(A) \succ (B)$. Therefore, the value maximizing decision maker will not choose (B). Analogous arguments but with $T_{CE}$ and $T_{AC}$ instead of $T_{BE}$ and $T_{AB}$, respectively, show that the value maximizing decision maker will not choose (C) either. Hence, in summary, we are left with only two cases: (A) and (E). Which of the two maximizes the value is determined by the equivalence relations:

$$
v_\lambda(x) + v_\lambda(y) + v_\lambda(z) \leq v_\lambda(x + y + z) \iff u(x) + u(y) - \lambda u(-z) \leq u(x + y + z) \iff T_{AE} \leq \lambda.
$$

This concludes the proof of the ‘max’ part.

**The ‘min’ part.** To prove the ‘min’ part, we only need to deal with (B)–(E), because we already know that $(A) \succ (D)$. Case (E) gives the minimal value when $T_{BE} \geq \lambda$, $T_{CE} \geq \lambda$, and $T_{DE} \geq \lambda$. If, however, there is at least one among $T_{BE}$, $T_{CE}$, and $T_{DE}$ not exceeding $\lambda$, then the minimum is achieved by one of (B), (C), and (D). To determine which of them and when is minimal, we employ simple algebra and obtain the equivalence relationships:

$$
\begin{align*}
(C) \succ (B) & \iff T_{BC} \leq \lambda \\
(D) \succ (B) & \iff T_{BD} \geq \lambda \\
(E) \succ (B) & \iff T_{BE} \leq \lambda
\end{align*}
\quad
\begin{align*}
(B) \succ (C) & \iff T_{BC} \geq \lambda \\
(D) \succ (C) & \iff T_{CD} \geq \lambda \\
(E) \succ (C) & \iff T_{CE} \leq \lambda
\end{align*}
\quad
\begin{align*}
(B) \succ (D) & \iff T_{BD} \leq \lambda \\
(C) \succ (D) & \iff T_{CD} \leq \lambda \\
(E) \succ (D) & \iff T_{DE} \leq \lambda
\end{align*}
$$

This finishes the proof of Theorem 3.4. □

**Theorem 3.5** Let the value function be $v_\lambda$, and let $x \geq 0 \geq y \geq z$. 

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Max: With the thresholds \( T_{BE} = T(x, y + z), T_{CE} = T(x + z, y), T_{DE} = T(x + y, z) \), and
\[
\begin{align*}
T_{BC} &= \frac{u(x) - u(x + z)}{u(-y - z) - u(-y)}, \\
T_{BD} &= \frac{u(x) - u(x + y)}{u(-y - z) - u(-z)}, \\
T_{CD} &= \frac{u(x + y) - u(x + z)}{u(-z) - u(-y)},
\end{align*}
\]

the following statements specify the four possible maximal values among (A)–(E):

- When \( T_{BE} \geq \lambda \), \( T_{BC} \geq \lambda \), and \( T_{BD} \geq \lambda \), then (B).
- When \( T_{CE} \geq \lambda \), \( T_{BC} \leq \lambda \), and \( T_{CD} \leq \lambda \), then (C).
- When \( T_{DE} \geq \lambda \), \( T_{BD} \leq \lambda \), and \( T_{CD} \geq \lambda \), then (D).
- When \( T_{BE} \leq \lambda \), \( T_{CE} \leq \lambda \), and \( T_{DE} \leq \lambda \), then (E).

Min: With the thresholds \( T_{AC} = T(x, z), T_{AD} = T(x, y) \),
\[
T_{AE} = \frac{u(x) - u(x + y + z)}{u(-y) + u(-z)},
\]

and the other ones defined in the ‘max’ part of this theorem, the following statements specify the four possible minimal values among (A)–(E):

- When \( T_{AC} \leq \lambda \), \( T_{AD} \leq \lambda \), and \( T_{AE} \leq \lambda \), then (A).
- When \( T_{AC} \geq \lambda \), \( T_{CD} \geq \lambda \), and \( T_{CE} \leq \lambda \), then (C).
- When \( T_{AD} \geq \lambda \), \( T_{CD} \leq \lambda \), and \( T_{DE} \leq \lambda \), then (D).
- When \( T_{AE} \geq \lambda \), \( T_{CE} \geq \lambda \), and \( T_{DE} \geq \lambda \), then (E).

**Proof.** Since \( -y \geq 0 \) and \( -z \geq 0 \), we have from inequality (2.3) that \( u(-y) + u(-z) \geq u(-(y + z)) \) and thus \( -\lambda u(-y) - \lambda u(-z) \leq -\lambda u(-(y + z)) \). The latter is equivalent to \( v_\lambda(y) + v_\lambda(z) \leq v_\lambda(y + z) \), which means that \((B) \not\supseteq (A)\).

The ‘max’ part. We have four cases (B)–(E) to deal with. To determine which of them and when is maximal among (B)–(E), we employ simple algebra and obtain the equivalence relationships:
\[
\begin{align*}
(B) \not\supseteq (C) &\iff T_{BC} \geq \lambda, \\
(B) \not\supseteq (D) &\iff T_{BD} \geq \lambda, \\
(B) \not\supseteq (E) &\iff T_{BE} \geq \lambda, \\
(C) \not\supseteq (B) &\iff T_{BC} \leq \lambda, \\
(C) \not\supseteq (D) &\iff T_{CD} \leq \lambda, \\
(C) \not\supseteq (E) &\iff T_{CE} \geq \lambda, \\
(D) \not\supseteq (B) &\iff T_{BD} \leq \lambda, \\
(D) \not\supseteq (C) &\iff T_{CD} \geq \lambda, \\
(D) \not\supseteq (E) &\iff T_{DE} \geq \lambda.
\end{align*}
\]
This finishes the proof of the ‘max’ part.
The ‘min’ part. To prove the ‘min’ part of the theorem, we verify the following four sets of orderings:

\[
\begin{align*}
(C) &\succ (A) \iff T_{AC} \leq \lambda, \\
(D) &\succ (A) \iff T_{AD} \leq \lambda, \\
(E) &\succ (A) \iff T_{AE} \leq \lambda, \\
(A) &\succ (C) \iff T_{AC} \geq \lambda, \\
(D) &\succ (C) \iff T_{CD} \geq \lambda, \\
(E) &\succ (C) \iff T_{CE} \leq \lambda.
\end{align*}
\]

This concludes the proof of the ‘min’ part and of Theorem 3.5 as well. \(\square\)

The following proposition is a simplification of Theorem 3.5 that we have achieved in the case of the value function \(v_{\lambda,\gamma}\).

**Proposition 3.1** Let the value function be \(v_{\lambda,\gamma}\) with \(\gamma \in (0,1)\), and let \(x \geq 0 \geq y \geq z\) with \(x + y + z > 0\). Furthermore, let the thresholds be same as in Theorem 3.5 but now with the function \(u(x) = x^\gamma\).

Max: The following statements specify the two possible maximal values among (A)–(E):

- When \(T_{BE} \geq \lambda\), \(T_{BC} \geq \lambda\), and \(T_{BD} \geq \lambda\), then (B).
- When \(T_{BE} \leq \lambda\), \(T_{CE} \leq \lambda\), and \(T_{DE} \leq \lambda\), then (E).

Min: The following statements specify the three possible minimal values among (A)–(E):

- When \(T_{AC} \leq \lambda\), \(T_{AD} \leq \lambda\), and \(T_{AE} \leq \lambda\), then (A).
- When \(T_{AC} \geq \lambda\), \(T_{CD} \geq \lambda\), and \(T_{CE} \leq \lambda\), then (C).
- When \(T_{AE} \geq \lambda\), \(T_{CE} \geq \lambda\), and \(T_{DE} \geq \lambda\), then (E).

**Proof.** To prove the ‘max’ part of the proposition, we need to show that the following two cases are impossible to realize:

- When \(T_{CE} \geq \lambda\), \(T_{BC} \leq \lambda\), and \(T_{CD} \leq \lambda\), then (C).
- When \(T_{DE} \geq \lambda\), \(T_{BD} \leq \lambda\), and \(T_{CD} \geq \lambda\), then (D).

We shall accomplish this task by establishing the inequalities:

\[
\begin{align*}
T_{CE} &< T_{BC}, \\
T_{DE} &< T_{BD}.
\end{align*}
\]
Indeed, when (3.11) holds, then the two conditions $T_{CE} \geq \lambda$ and $T_{BC} \leq \lambda$ cannot be fulfilled simultaneously, and when (3.12) holds, then the two conditions $T_{DE} \geq \lambda$ and $T_{BD} \leq \lambda$ cannot be fulfilled simultaneously.

To prove inequality (3.11), we employ simple algebra and obtain that the inequality is equivalent to

$$T(x + z, y) < T(x, y + z),$$

which can be rewritten as $T_{CE} < T_{BE}$. Since $u(x) = x^\gamma$, bound (3.13) is equivalent to

$$\left(\frac{x + z}{-y}\right)^\gamma - \left(\frac{x + z}{-y} - 1\right)^\gamma < \left(\frac{x}{-y - z}\right)^\gamma - \left(\frac{x}{-y - z} - 1\right)^\gamma.$$

To prove that the latter holds, we observe that $v^\alpha - (v - 1)^\alpha$ is a strictly decreasing function in $v$ whenever $\gamma \in (0, 1)$, and that the bounds

$$\frac{x + z}{-y} > \frac{x}{-y - z} > 1$$

hold because $x + y + z > 0$. This proves inequality (3.11).

The proof of inequality (3.12) is analogous. First, simple algebra shows that the inequality is equivalent to

$$T(x + y, z) < T(x, y + z),$$

which is $T_{DE} < T_{BE}$. Bound (3.14) is equivalent to

$$\left(\frac{x + y}{-z}\right)^\gamma - \left(\frac{x + y}{-z} - 1\right)^\gamma < \left(\frac{x}{-y - z}\right)^\gamma - \left(\frac{x}{-y - z} - 1\right)^\gamma.$$

Since $v^\alpha - (v - 1)^\alpha$ is strictly decreasing in $v$, to complete the proof of inequality (3.12), we only note that

$$\frac{x + y}{-z} > \frac{x}{-y - z} > 1.$$

The ‘max’ part of Proposition 3.1 is finished.

To prove the ‘min’ part of the proposition, we shall show that the following case is impossible:

- When $T_{AD} \geq \lambda$, $T_{CD} \leq \lambda$, and $T_{DE} \leq \lambda$, then (D).

For this, we establish the inequality

$$T_{AD} < T_{DE},$$

which is equivalent to

$$\left(\frac{x}{-y}\right)^\gamma - \left(\frac{x}{-y} - 1\right)^\gamma < \left(\frac{x + y}{-z}\right)^\gamma - \left(\frac{x + y}{-z} - 1\right)^\gamma.$$
It now remains to observe that
\[
\frac{x}{-y} > \frac{x + y}{-z} > 1.
\]
This finishes the proof of the ‘min’ part, and thus of Proposition 3.1 as well. □

We conclude this section with a corollary to Theorem 3.5 based on a different value function than any one noted so far. It is a somewhat ‘pathological’ results, but it helps us to better see and understand the complexities and challenges associated with multiple exposure units.

**Corollary 3.1** Let the value function be \(v_\lambda\) with \(u(x) = 1 - \exp\{-\varrho x\}\) and parameter \(\varrho > 0\), and let \(x \geq 0 \geq y \geq z\). Then all the thresholds noted in the ‘max’ part of Theorem 3.5 are equal to \(T_{\text{max}} = \exp\{-\varrho(x+y+z)\}\). Hence, the following statements specify the four possible maximal values among (A)–(E): when \(T_{\text{max}} \geq \lambda\), then (B); when \(T_{\text{max}} = \lambda\), then (C) and (D); and when \(T_{\text{max}} \leq \lambda\), then (E). Moreover, when \(T_{\text{max}} = \lambda\), then all cases (B)–(E) give the same value \(1 - \exp\{-\varrho(x+y+z)\}\).

### 4 Illustration

Numerical examples that we shall present in the second subsection below are designed to illustrate our earlier theoretical results, especially their optimality. Though we have not made a special effort to place the examples into a real life context, one can easily do so within, say, the context of marketing (e.g., Drumwright, 1992; Heath et al., 1995; and references therein).

#### 4.1 A marketing illustration

Following Drumwright (1992), let \(R\) denote the reservation price of a product, which is the largest price that the consumer is willing to pay in order to acquire the product. Furthermore, let \(M\) be the market price of the product. The consumer buys the product if the consumer surplus is non-negative: \(R - M \geq 0\).

Assume that a company is manufacturing two products, \(A\) and \(B\). Let their reservation prices be \(R_A = 21\) and \(R_B = 10\), and the market prices \(M_A = 15\) and \(M_B = 15\). We would predict following the basic economic theory that the consumer would buy only the product \(A\), because the consumer surplus is positive only for this product. However, bundling can make the consumer also buy the product \(B\), thus increasing the company’s revenue. Namely, suppose that a bundle of the two products \(A\) and \(B\) sells at a price of 30. Then, according to mental accounting, the consumer will buy the bundle. To demonstrate this rigorously, assume that the consumer is loss averse in the sense that \(\lambda \geq 1\).
Using our adopted terminology, the two experiences (consumer surpluses) corresponding to $A$ and $B$ are $x = R_A - M_A = 6$ and $y = R_B - M_B = -5$, respectively. The total experience is positive: $x + y = 1$. The value of the individually purchased products is $v_\lambda(6) + v_\lambda(-5)$. If they are bundled, then the consumer surplus is $(R_A + R_B) - (M_A + M_B) = 31 - 30 = 1$, and the value is $v_\lambda(1)$. Theorems 2.2 and 2.3 imply $v_\lambda(1) \geq v_\lambda(6) + v_\lambda(-5)$ because $\lambda \geq 1$ and $T(x) \equiv T(6, -5) \leq 1$, thus implying that $T(x) \leq \lambda$, which means ‘integration’ for the value maximizing decision maker. Hence, the company is better off when the two products are bundled: the revenue is 30 by selling both products as a bundle, whereas the revenue is just 15 when the two products are sold separately, because in the latter case the consumer buys only the product $A$.

In the case of more than two products, the bundling strategy becomes much more complex, with five possibilities (A)–(E) in the case $n = 3$ considered above. We shall next illustrate our earlier developed theory.

4.2 Examples

In the following examples we use the $S$-shaped value function (al-Nowaihi et al., 2008)

$$v_{\lambda, \gamma}(x) = \begin{cases} x^\gamma & \text{when } x \geq 0, \\ -\lambda(-x)^\gamma & \text{when } x < 0. \end{cases} \quad (4.1)$$

Obviously, $v_{\lambda, \gamma} = v_\lambda$ with $u(x) = x^\gamma$.

The following two numerical examples illustrate the validity of principles P1 and P2.

**Example 4.1 (illustrating principle P1)** Let the value function be $v_{\lambda, \gamma}$ with the parameters $\lambda = 2.25$ and $\gamma = 0.88$. Suppose that we have three exposure units with positive experiences 5, 10, and 20. Principle P1 suggests segregating them, and this is mathematically confirmed by the inequality: $v_{\lambda, \gamma}(\sum x_k) = 22.8444 < \sum v_{\lambda, \gamma}(x_k) = 25.6683$. (We use $\sum$ instead of $\sum_{k=1}^3$ to simplify notation.) Our general results say that the value maximizing decision maker prefers segregating any number of positive exposures. □

**Example 4.2 (illustrating principle P2)** Let the value function be $v_{\lambda, \gamma}$ with the parameters $\lambda = 2.25$ and $\gamma = 0.88$. Suppose that we have three exposure units with negative experiences $-5$, $-10$, and $-20$. Principle P2 suggests integrating them, and this is confirmed by the inequality: $v_{\lambda, \gamma}(\sum x_k) = -51.3999 > \sum v_{\lambda, \gamma}(x_k) = -57.7537$. Our general results say that the value maximizing decision maker prefers integrating any number of negative exposures. □

The following two examples show that principles P3 and P4 can be violated.
Example 4.3 (illustrating principle P3) Let the value function be $v_{\lambda, \gamma}$ with the parameters $\lambda = 2.25$ and $\gamma = 0.88$. Suppose that we have three exposure units with mixed experiences $-0.5$, $10$, and $20$, whose total (positive) experience is $\sum x_k = 29.5$. Principle P3 would suggest integrating the exposure units into one, but the following inequality implies the opposite: $v_{\lambda, \gamma}(\sum x_k) = 19.6537 < \sum v_{\lambda, \gamma}(x_k) = 20.3239$. In fact, we see from our theoretical analysis of the case $n = 3$ that neither complete segregation nor complete integration of three (or more) experiences with mixed exposures may lead to a maximal value, which may be achieved only by a partial integration and segregation. □

Example 4.4 (illustrating principle P4) Let the value function be $v_{\lambda, \gamma}$ with the parameters $\lambda = 2.25$ and $\gamma = 0.88$. Suppose that we have three exposure units with mixed experiences $0.5$, $-10$, and $-20$, whose total (negative) experience is $\sum x_k = -29.5$. Principle P4 suggests segregating the exposure units, but the following inequality says the opposite: $v_{\lambda, \gamma}(\sum x_k) = -44.2207 > \sum v_{\lambda, \gamma}(x_k) = -47.9361$. Our theory developed above says that neither complete segregation nor complete integration may lead to a maximal value when $n \geq 3$. □

The following two examples illustrate Theorem 2.2 in the case of three exposure units and assuming that the decision maker is given only two options: either integrate all exposure units or keep them segregated.

Example 4.5 (illustrating Theorem 2.2) Let $x_1 = 25$, $x_2 = 10$, and $x_3 = -0.5$, with the positive total sum $x_1 + x_2 + x_3 = 34.5$. Let the value function be $v_{\lambda, \gamma}$ with $\gamma = 0.88$. The threshold $T(x) = 3.7149$. Thus, facing the dilemma of integrating or segregating all exposure units (we are not dealing with any partial integration or partial segregation in this example), the decision maker prefers segregating them when $\lambda \leq 3.7149$ and integrating them when $\lambda \geq 3.7149$. An additional illustration is provided in Table 4.1. □

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$v_{\lambda, \gamma}(\sum x_k)$</th>
<th>$\sum v_{\lambda, \gamma}(x_k)$</th>
<th>Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>22.557</td>
<td>24.3039</td>
<td>Segregate</td>
</tr>
<tr>
<td>1.50</td>
<td>22.557</td>
<td>23.7605</td>
<td>Segregate</td>
</tr>
<tr>
<td>4.00</td>
<td>22.557</td>
<td>22.4021</td>
<td>Integrate</td>
</tr>
</tbody>
</table>

Table 4.1: Total segregation vs total integration; with $T(x) = 3.7149$.

Example 4.6 (illustrating Theorem 2.2) Let $x_1 = -6$, $x_2 = -5$, and $x_3 = 10$, with the negative total sum $x_1 + x_2 + x_3 = -1$. Let the value function be $v_{\lambda, \gamma}$ with $\gamma = 0.88$. The threshold $T(x) = 0.9529$, and thus, facing the dilemma of integrating or segregating...
all exposure units (we are not dealing with any partial integration or partial segregation in this example), the decision maker prefers segregating when $\lambda \leq 0.9529$ and integrating when $\lambda \geq 0.9529$. An additional illustration is provided in Table 4.2. □

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$v_{\lambda, \gamma}(\sum x_k)$</th>
<th>$\sum v_{\lambda, \gamma}(x_k)$</th>
<th>Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>-0.50</td>
<td>3.1056</td>
<td>Segregate</td>
</tr>
<tr>
<td>0.98</td>
<td>-0.98</td>
<td>-1.1961</td>
<td>Integrate</td>
</tr>
<tr>
<td>1.50</td>
<td>-1.50</td>
<td>-5.8558</td>
<td>Integrate</td>
</tr>
</tbody>
</table>

Table 4.2: Total segregation vs total integration; with $T(x) = 0.9529$.

**Example 4.7 (illustrating Theorem 3.2)** Assume that the value function is $v_{\lambda, \gamma}$ with $\gamma = 0.88$. With the experiences $x = 5$, $y = 3$, and $z = -2$, we have $T_{AC} = 0.8109$ and $T_{ED} = 0.7575$. Hence, the following statements hold:

Max: When $\lambda \leq 0.8109$, then (A) is maximal, and when $0.8109 \leq \lambda$, then (C) is maximal.

Min: When $\lambda \leq 0.7575$, then (E) is minimal, and when $0.7575 \leq \lambda$, then (D) is minimal.

**Example 4.8 (illustrating Theorem 3.3)** Assume that the value function is $v_{\lambda, \gamma}$ with $\gamma = 0.88$. With the experiences $x = 10$, $y = 1$, and $z = -2$, we have $T_{AC} = 0.7349$, $T_{BD} = 0.7897$, and $T_{BE} = 0.6717$. Hence, the following statements hold:

Max: When $\lambda \leq 0.7349$, then (A) is maximal, and when $0.7349 \leq \lambda$, then (C) is maximal.

Min: When $\lambda \leq 0.6717$, then (E) is minimal, when $0.6717 \leq \lambda \leq 0.7897$, then (B) is minimal, and when $0.7897 \leq \lambda$, then (D) is minimal.

**Example 4.9 (illustrating Theorem 3.4)** Assume that the value function is $v_{\lambda, \gamma}$ with $\gamma = 0.88$. With the experiences $x = 4$, $y = 3$, and $z = -5$, we have $T_{AE} = 0.8404$, $T_{BD} = 0.9446$, $T_{CE} = 0.7890$, and $T_{CB} = 0.9014$. Hence, the following statements hold:

Max: When $\lambda \leq 0.8404$, then (A) is maximal, and when $0.8404 \leq \lambda$, then (E) is maximal.

Min: When $\lambda \leq 0.7890$, then (E) is minimal, when $0.7890 \leq \lambda \leq 0.9014$, then (C) is minimal, when $0.9014 \leq \lambda \leq 0.9446$, then (B) is minimal, and finally when $0.9446 \leq \lambda$, then (D) is minimal.

**Example 4.10 (illustrating Proposition 3.1)** Assume that the value function is $v_{\lambda, \gamma}$ with $\gamma = 0.88$. With the experiences $x = 36$, $y = -2$, and $z = -14$, we have $T_{BE} = 0.8243$, $T_{AC} = 0.8074$, and $T_{CE} = 0.6636$. Hence, the following statements hold:
Max: When $\lambda \leq 0.8243$, then $B$ is maximal, and when $0.8243 \leq \lambda$, then $E$ is maximal.

Min: When $\lambda \leq 0.6636$, then $E$ is minimal, when $0.6636 \leq \lambda \leq 0.8074$, then $C$ is minimal, and when $0.8074 \leq \lambda$, then $A$ is minimal.

4.3 An experiment

We have seen from our mathematical considerations that the ‘classical’ hedonic editing hypotheses (principles P1–P4) in the case of multiple exposure units may not always hold. In order to test this fact in practice, we have run an experiment.

4.3.1 Empirical evidence

Some studies have tested the hypotheses (principles P1–P4) empirically for two exposure units. For example, Thaler (1985) evaluates the happiness of subjects facing two different scenarios, which result in four possible types of outcomes. While this experiment supports the hedonic editing models, two experiments by Thaler and Johnson (1990), and Linville and Fischer (1991) on temporal spacing of outcomes reject integration of losses and develop some variations of the initial model: quasi-hedonic editing model, and renewable resources model. There are several works focussing on different aspects of mental accounting, such as frame sensitivity (Heath et al., 1995) and hedonic limen (Morewedge et al., 2007), or focussing on other applications such as investor decisions (Lim, 2006) and tax returns (Moreno et al., 2007). Furthermore recent studies by Wu and Markle (2008), and Jarnebrant et al. (2009) test Thaler’s principles in the case of mixed gambles. All these studies focus on the case of two outcomes, even if they sometimes deal with multiple outcomes; we are not aware of any experiment that would explicitly test mental accounting principles in the case of more than two outcomes.

4.3.2 Experimental design

In the literature we find two ways for considering mental accounting for sure outcomes. The first one, which is in the line of Thaler (1985), proposes evaluating happiness of two fictitious situations in which outcomes are divided into two events (segregation) or combined into one single event (integration). The second one, due to Thaler and Johnson (1990), and Linville and Fisher (1991), is based on timing separation of events: subjects are asked if they prefer an outcome at a single moment (integration) or at different moments (segregation).

In our current experiment we use the second approach and thus assume that we are testing the hedonic editing hypotheses by eliciting preferences of subjects over the timing of events. We use only hypothetical choices, and there are several reasons for supporting this choice.
For example, a serious issue would be a practical implementation of an experiment during which subjects would possibly suffer actual monetary losses. Not surprisingly therefore, almost all papers in the area deal with hypothetical choices, even in the case of gambles, and the paper by Thaler and Johnson (1990) which introduces monetary incentives actually finds no significant differences between treatments with hypothetical and monetary choices. We can also find papers arguing that there are no major quantitative differences between hypothetical choices and real monetary choices (Camerer and Hogarth, 1999), and thus the question of monetary incentives should not be the main aspect of the problem (Read, 2005).

The experimental design that we have used in our research is based on the following task (see Appendix for details): subjects had to fill in a Likert scale of preferences (from -2 to +2), facing two possible choices with timing separation of sure events. Different patterns of choice have been used: gains, losses, mixed gains, mixed losses, and ties. The number of outcomes varied: three and five. In total, subjects had to answer fourteen questions.

### 4.3.3 Results

Our experiments was conducted in November 2010, with 100 students at the North-East Normal University (Changchun), 15 students at Hong Kong Baptist University, and 55 students at the University of Paris 1.

First, we pool the actual data in order to test if the hedonic hypotheses are robust to multiple outcomes. Then we check if the number of outcomes (three or five) has had an impact on the results. Finally, we explored inter-cultural difference by comparing the results from Changchun, Hong Kong, and Paris. Table 4.3 contains our findings. We see that the mental accounting predictions are not supported by three of our five patterns: gains, losses, and mixed losses. The mental accounting rules seem to only work for mixed gains and

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<th>Preferences: Actual (Prediction)</th>
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<td>51.5</td>
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Table 4.3: Results for pooled data: consistency with mental accounting (P1–P4) predictions reported in percentages; means of the preference values with statistically significant departures from zero (i.e. indifference) noted in %; and actual and predicted preferences.
ties. The behaviour of the subjects therefore suggests that our earlier note that the mental accounting rules as they are cannot be easily extend to the case of multiple outcomes. Specifically, our results show that the subjects preferred integrating gains, were indifferent between integrating and segregating losses, preferred integrating small losses into big gains, integrating small gains into losses, and integrating gains and losses in the case of ties.

It is also interesting to know if the number of outcomes has played a role. Table 4.4 presents the results for three and five outcomes. We find some behavioral differences between the two cases, but only the level of preferences in the case of mixed gains is statistically significant: subjects have stronger preferences for integration in the case of three outcomes (p-value of the mean differences is significant at 10%).

We have also compared results between the three locations: Changchun, Hong Kong, and Paris. Table 4.5 summarizes these results. The comparison offers robust differences. The two most important are the cases of gains and losses. For gains, the French students preferred integrating gains, and students in Changchun seem to have been indifferent. The values of preferences are significantly different (p-value of the mean differences is significant at 5%). For losses, we find reverse preferences: the French students preferred segregating losses, whereas students in Changchun seem to have preferred integrating losses (p-value of the mean differences is significant at 1%). The latter implies that the percentage of answers violating mental accounting rules is significantly higher for the French students (p-value of the percentage differences is significant at 5%). Note that the results of the Hong Kong students seem to follow a different pattern, with a lot of indifferences, but since the number

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Table 4.4: Results for three and five outcomes
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<td>50.9</td>
<td>−0.25 5%</td>
<td>Segregation (Integration)</td>
</tr>
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</table>

Table 4.5: Results for Changchun, Hong Kong, and Paris data

of subjects is small (15 students), we are unable to conclude their statistical significance.

The experimental results show that, when more than two outcomes are involved, choices can violate Thaler’s principles. Our empirical results do not support principles P1 and P2. This is somewhat surprising because, theoretically, both results do not depend on the number of outcomes that individuals are exposed to: using Petrović’s inequality, we have shown that individuals should ideally prefer segregating gains or integrating losses, but this has not manifested in our experiments. Thaler’s principles seem to have worked well only for mixed gains (principle P3) and ties. For mixed losses (principle P4) the results also differ from Thaler’s predictions.

As we see from our theoretical analysis, when more than two outcomes are involved, the optimal choice becomes increasingly complex, with partial integration or partial segregation producing optimal results. This experiment shows that further and more exhaustive experimental studies of mental accounting for multiple outcomes would be of interest.
5 Conclusions

Our theoretical study has shown that within the class of value functions specified by the prospect theory, the validity of Thaler’s principles can be established rigorously in the case of only non-negative experiences, or only non-positive experiences, and irrespectively of the number of exposure units. When exposure units carry both negative and positive experiences, then the principles may break down. Our theory provides a complete solution to the integration/segregation problem in the case of three exposure units and demonstrates in particular that the transition from two to three, or more, exposure units increases the complexity of decisions enormously, thus showing the challenges that the decision maker encounters when dealing with multiple exposure units.

In addition to our theoretical results of the previous sections, there might also be situations when the decision maker needs – for one reason or another – to use partial integration of, say, non-negative experiences. In such cases, a generalization of Lim’s (1971) inequality (cf. Kuczma, 2009) plays a decisive role. Namely, for any continuous and concave function \( v : [0, \infty) \to \mathbb{R} \), and for any triplet \((x_1, x_2, x_3)\) of non-negative real numbers such that \( x_3 \geq x_1 + x_2 \), we have that

\[
v(x_1) + v(x_2 + x_3) \leq v(x_1 + x_2) + v(x_3).
\]

This inequality implies in particular that if we have three exposure units with positive experiences and if for some reason we can only integrate two of them, then in order to decide whether, say, \( x_2 \) should be integrated with \( x_1 \) or \( x_3 \), the value maximizing decision maker would need to verify the condition \( x_3 \geq x_1 + x_2 \), and if it holds, then the decision maker should integrate \( x_2 \) with \( x_1 \), leaving \( x_3 \) segregated.

Acknowledgements

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References


Appendix

Our instructions that we have given to students are reproduced in the first subsection below. The second subsection contains a list of questions that we have given to the students. The third subsection contains raw data in the form of percentages for each scenario and for each level on the Likert scale (from -2 to +2).

The instructions of the experiment

Below you find two scenarios, titled A and B, in which some events occur. The events are the same in their total value but they are either aggregated (scenario B) on one single day or segregated on different consecutive days (scenario A). You are asked to rate your level of preferences between the two scenarios on a scale from -2 (strongly prefer the integrated offer, i.e., scenario B) to +2 (strongly prefer the segregated offer, i.e., scenario A).

Note: Having the events occur together does not imply that they occur sooner or later than if they were apart. That is not the question. You are only asked to judge whether it is better to have the events separated or together.

(In order to rate their level of preferences for integration (-2 or -1), segregation (+1 or +2) or indifference (value 0), students had to fill the following Likert scale.)

Please rate your level of preferences from -2 (scenario B) to +2 (scenario A)

| -2 | -1 | 0 | +1 | +2 |

Table 5.1: Likert scale of preferences
The list of questions

**THREE OUTCOMES**

1. seg) Three gains of $50 each  
    int) Gain of $150  
2. seg) Three losses of $50 each  
    int) Loss of $150  
3. seg) Two gains of $30 each and a loss of $10  
    int) Gain of $50  
4. seg) Two gains of $25 each and a loss of $100  
    int) Loss of $50  
5. seg) Two gains of $10 each and a loss of $20  
    int) No gains and no losses  
6. seg) Gain of $50 and two losses of $10 each  
    int) Gain of $30  
7. seg) A gain of $50 and two losses of $50 each  
    int) Loss of $50  
8. seg) A gain of $50 and two losses of $25 each  
    int) No gains and no losses

**FIVE OUTCOMES**

9. seg) Five gains of $20 each  
    int) A gain of $100  
10. seg) Three losses of $10 each and two losses of $35 each  
    int) A loss of $100  
11. seg) Three gains of $40 each and two losses of $10 each  
    int) A gain of $100  
12. seg) Two gains of $25 each and three losses of $50 each  
    int) A loss of $100  
13. seg) Three gains of $40 each and two losses of $60 each  
    int) No gains and no losses  
14. seg) Two gains of $60 each and three losses of $40 each  
    int) No gains and no losses
### Summary statistics (in percentages)

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