The Dynamics of Immigrant Earnings (Very Preliminary and Incomplete)

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Abstract

We develop a model that accounts for several stylized facts about the evolution of immigrant earnings. First, it implies that new immigrants learn less than natives with a similar level of schooling. Second, it implies that the earnings gap narrows over time. Finally, the model also implies that the size of earnings gap between new immigrants and natives as well as the rate at which the gap decreases over time, depend on the individual's country of origin.

In addition to establishing some theoretical results, we develop a calibrated version of the model and we show that it is broadly consistent with the evidence.

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1. Introduction

It is a well established fact that earnings of immigrants and natives are not equal even when they have similar levels of schooling. Moreover, the evidence suggests that the dynamics of earnings over the life cycle are different for natives and migrants. We summarize the evidence in the following "facts"[1]:

- **Migrants vs. Stayers**. Migrants and stayers with the same schooling have different age-earnings profiles. Upon migration, migrants earn more and their earnings grow faster than the earnings of stayers.

- **Migrants vs. Natives**. Even after controlling for schooling, migrants earn initially less than natives. However, they show steeper age earnings profiles. The gap in initial earnings is negatively related to the level of GDP per capita in the country of origin.

- **Option to Migrate**. There is limited but suggestive evidence that increases in the probability that individuals be allowed to migrate to a high wage location has a positive (and large) effect on investments in human capital of the whole eligible population.

In addition to those "facts" there is some evidence that suggest that:

- **Scale**: Migrants tend to be drawn from the ranks of the high skilled.

- **Positive Selection**: The mix of skilled/unskilled (immigrants) in the destination country is higher than in the sending country.

[1] In section XXX we provide references and more precise definitions of some terms that can reasonably be viewed as somewhat vague.

- **Positive Sorting**: The higher the wage in the receiving location the larger the ratio of skilled to unskilled immigrants.

This paper presents a version of the human capital model originally developed by Ben Porath augmented with the decision to migrate to understand the forces that can account for these observations. Our work can be viewed as a dynamic version —with endogenous accumulation of human capital— of the more standard Roy model that is the workhorse of the migration related literature.

First, we explore the theoretical implications of the model in the case of unanticipated migration. We show that, at least qualitatively, the predictions of the theory match the evidence about the difference in age earnings profiles between migrants and stayers. In this case the key mechanism is that migrants and stayers face different prices and, hence, they make different decisions in terms of on-the-job training efforts that result in a steeper age-earnings profile for the individuals in the high wage location.

We then explore the implications for the differences between migrants and natives. A key insight is that in the model if two individuals chose to acquire the same amount of schooling in different economic environments, then they cannot be identical. Moreover, in a model that allows education to be two dimensional —with a quantity dimension as measured by years of schooling, and a quality dimension that we view as a better measure of human capital— it is optimal for the individual in the low wage location to choose a lower quality of schooling. This, in turn, implies that he has lower human capital than a native at the time of migration. These two results explain both the lower initial earnings —because initial human capital is lower— and the steeper age-earnings profiles —because the migrant must have had higher innate ability to choose the same years of schooling in a lower return environment. Moreover, since the "quality" of human capital holding years of schooling constant varies positively, the model also accounts for the observation that migrants from poorer locations earn less than migrants from higher income regions.

Our view implies that differences in earnings are not a good measure of differences in TFP across countries. Thus, our approach contrasts with the more standard approach, as exemplified by Hendricks (2002), that views differences in earnings for migrants and natives with the same schooling as direct measures of TFP. This view is silent about the differences in age-earnings profiles.

We study the impact of facing individuals with a known probability of migration in a context in which migration is costly in terms of goods and human capital. Our main result suggests that whether there is positive or negative selection depends in a somewhat complicated manner on the "size" of the stock of human capital lost, the differences in wage rates between the sending and receiving locations and the age of the potential migrant.

We then use a calibrated version of the model to understand whether human capital and selection can account for the evidence on the earnings of immigrants in the U.S. Even though our exercise is very preliminary, we find the results encouraging as the model seems consistent with the data.

2. Theoretical Results

In this section we describe the basic model, characterize its solution, and describe the implications for output per worker using the exogenously specified demographic structure.

2.1. Unexpected Migration: Individual Decision Problem

Here we present a sketch of the model. A more thorough treatment can be found in Manuelli and Seshadri (2013). The representative individual maximizes the present discounted value of net income. We assume that each agent lives for T periods and retires at age $R \leq T$. The maximization problem is

$$\begin{aligned} \max_{x_s, x_w, x_E, n, s} \int_6^R & e^{-r(a-6)}[(1-\tau)wh(a)(1-n(a)) \\ & -(\aleph_{[n(a)=1]}p_s x_s(a) + (1-\aleph_{[n(a)=1]})p_w x_w(a))]da - p_E x_E \end{aligned} \tag{2.1}$$

subject to

$$\dot{h}(a) = z_h[n(a)h(a)]^{\gamma_1}x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [6, R), \tag{2.2}$$

and

$$h(6) = h_E = h_B x_E^{\upsilon} \tag{2.3}$$

with h_B given.[2] Here, $\aleph_{[n(a)=1]}$ is an indicator function that takes the value one whenever the individual allocates 100% of his time to schooling. Even though the notation is somewhat cumbersome, it is necessary to allow for the price of the inputs used in the production of human capital to vary depending on whether the individual is in or out of school.

Equations (2.2) and (2.3) correspond to the standard human capital accumulation model initially developed by Ben-Porath (1967). This formulation allows for both market goods, $x_j(a)$ $j \in \{s, w\}$, and a fraction $n(a)$ of the individual's

[2] The assumption of linear utility is without loss of generality. It can be shown that the solution to the income maximization problem is also the solution to a utility maximization problem when the number of children is given, parents have a bequest motive, and bequests are unconstrained. For details, see Manuelli and Seshadri (2013).

human capital, to be inputs in the production of human capital. We assume that in the "schooling period," which we identify with the length of time that the individual optimally chooses $n(a) = 1$, the price of market inputs used in the production of human capital is p_s, while in the working period, the unit price of market inputs that increase human capital is p_w. Investments in early childhood[3], which we denote by x_E (e.g. medical care, nutrition and development of learning skills), determine the level of each individual's human capital at age 6, $h(6)$, or h_E for short.[4] Our formulation captures the idea that nutrition and health care are important determinants of early levels of human capital, and those inputs are, basically, market goods.[5]

There are two important features of our formulation. First, we assume that the human capital accumulation technology is the same during the schooling and the training periods. Second, we assume that the market inputs used in the production of human capital —$x_j(a)$, $j \in \{s, w\}$— are privately purchased. In the case of the post-schooling period, this is not controversial. However, this is less so for the schooling period. Here, we take the 'purely private' approach as a first pass.[6] In fact, for our results to go through, it suffices that, at the margin, individuals pay for the last unit of market goods allocated to the formation of

[3]The emphasis on early childhood as one of the important determinants of human capital formation follows Carneiro, Cunha and Heckman (2003) who also specify a production technology in which goods and time invested by parents affect children's human capital.

[4]It should be made clear that market goods ($x(a)$ and x_E) are produced using the same technology as the final goods production function. Hence the production function for human capital is more labor intensive than the final goods technology.

[5]It is clear that parents' time is also important. However, given exogenous fertility, it seems best to ignore this dimension. For a full discussion see Manuelli and Seshadri (2009). We allow for the possibility that the price of the inputs needed to produce early childhood human capital, p_E in our notation, is not equal to the price of general consumption.

[6]An alternative explanation is that Tiebout like arguments effectively imply that public expenditures on education play the same role as private expenditures. The truth is probably somewhere in between.

human capital.

The full solution to the income maximization problem is presented in the Appendix. The solution to the problem is such that $n(a) = 1$, for $a \leq 6 + s$. Thus, we identify s as *years of schooling*. In what follows we assume that the economy is at the steady state and, hence, that the relevant prices are fixed and given by the marginal products.[7]

In what follows we describe the implications of this simple investment model for the age earnings profiles of "similar," in the sense that they have exactly the same years of schooling, individuals that differ in their destination countries (i.e. we compare migrants with stayers) and in the environment in which they acquired their education (i.e. we compare migrants with natives).

2.1.1. Migrants vs. Stayers

Consider now two individuals in the home country who have completed their desired level of schooling and have exactly the same amount of human capital (and equal schooling) at age a_m. We identify the "home" country with a vector of prices: after tax wages, $(1-\tau)w$, and the price of market inputs used in on-the-job training, p_w. As a matter of notation we use * to indicate the same variables in the destination location.

The results in Manuelli and Seshadri (2013) show that the amount of human capital accumulated after age a_m —the age of migration— is a function of the

[7] In this version we abstract from financial constraints and assume that financial or family markets do not keep individuals from making efficient investments in human capital. Even though there is some evidence that financial constraints do not seem to be very important in influencing schooling decisions in the U.S. it is less clear that this is a resonable assumption in the case of very poor countries. However, it seems that a natural first step should be to understand the workings of the model in the absence of frictions other than those imposed by the technology.

prices prevailing in the location in which investment takes place. Then, the level of human capital at age a is

$$h(a) = e^{-\delta_h(a-a_m)}[h(a_m) + \frac{r+\delta_h}{\gamma_1}\left(\frac{\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} \tag{2.4}$$

$$\int_{a_m}^{a} e^{-\delta_h(t-a_m)} m(t)^{\frac{\gamma}{1-\gamma}} dt, \qquad a \geq a_m,$$

where

$$m(a) = 1 - e^{-(r+\delta_h)(R-a)},$$

To simplify the presentation, let's assume that $(1-\tau^*)w^* = (1+\varepsilon)((1-\tau)w)$, $\varepsilon > 0$, and $p_w^* = p_w$.[8]. Let $h_m(a)$ and $h_s(a)$ denote the human capital level of the migrant and the stayer respectively. Define the (adjusted) difference in human capital by

$$\Delta_h^s(a, a_m; \varepsilon) = e^{\delta_h(a-a_m)}[h_m(a) - h_s(a)].$$

It follows that

$$\Delta_h^s(a, a_m; \varepsilon) = [(1+\varepsilon)^{\gamma_2/(1-\gamma)} - 1]\frac{r+\delta_h}{\gamma_1}$$

$$\left(\frac{\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} \int_{a_m}^{a} e^{-\delta_h(t-a_m)} m(t)^{\frac{\gamma}{1-\gamma}} dt.$$

In order to describe the effects of migration consider first the case in which the retirement age —R in the model— is the same in the origin and destination countries. In this case:

1. The larger the difference between $(1-\tau^*)w^*$ and $((1-\tau)w)$ —i.e. the larger

[8] The extension to the case in which the input prices vary is straightforward.

the value of ε— the larger the human capital differentials. Moreover, if $\gamma_2 > (1-\gamma)$ (which holds in all our parameterizations), $\Delta_h^s(a, a_m; \varepsilon)$ is convex in ε which implies that wage differentials between origin and destination locations result in more than proportional human capital differentials.

2. The function $\Delta_h^s(a, a_m; \varepsilon)$ is increasing in a for $a = a_m$ and decreasing for $a = R$. Moreover, there exists an $\hat{a} < R$ that maximizes $\Delta_h^s(a, a_m; \varepsilon)$ over a. This implies that, upon migration, the human capital gap between migrants and stayers increases up to a point and then decreases.

3. It can be shown that $\Delta_h^s(a, a_m; \varepsilon)$ is concave in a whenever it is increasing (and at $\hat{a}$) and possibly convex for $a = R$. This implies that the rate of change in the human capital gap between migrants and stayers is large for initial ages and it gets smaller as both individuals near retirement.

4. The function $\Delta_h^s(a, a_m; \varepsilon)$ is decreasing in a_m. Thus, the older the age of migration, the smaller the human capital gap between migrants and stayers.

5. It can be shown that if migrants expect to retire earlier, this difference reduces the human capital gap with stayers.

6. Since the basic Ben-Porath model implies that the level of schooling —given all prices— is increasing in ability, z_h in the model, human capital differences are larger for those with higher levels of schooling.

One important message from these results is that the differences between the levels of human capital of migrants and stayers depend in a non-linear way on the differences in environments. Whether these non-linearities are important is a quantitative question that we will explore later.

In the model earnings of a worker are given by

$$y(a; w, p_w) = (1-\tau)wh(a)(1-n(a)) - p_w x_w(a).$$

In the appendix we show that optimal behavior implies that given the level of human capital at the time of migration, $h(a_m)$, the difference in the earnings of a migrant and a stayer defined as

$$\Delta_y^s(a, a_m; \varepsilon) = e^{\delta_h(a-a_m)}[y(a; w^*, p_w^*) - y(a; w, p_w)]$$

is given by

$$\Delta_y^s(a, a_m; \varepsilon) = \varepsilon e^{\delta_h(a-a_m)} y(a; w, p_w) \tag{2.5}$$
$$+[(1+\varepsilon)^{\gamma_2/(1-\gamma)} - 1] \left(\frac{\gamma_1}{r+\delta_h} \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2} \right)^{1/(1-\gamma)} \frac{1}{\gamma_1} \theta(a, a_m),$$

where the function $\theta(a, a_m)$ is defined in the Appendix. There, we show that

$$\theta(a_m, a_m) < 0,\ \theta(R, a_m) > 0 \text{ and} \frac{\partial \theta}{\partial a}(a, a_m) > 0.$$

Equation (2.5) summarizes the implication of the model for the differences in (adjusted) age earnings profile.[9] These differences have two components. First, the migrant's earnings at any age have to be scaled up by the differences in wage rates. This is the standard implication of exogenous (or learning-by-doing with indivisible labor) human capital models. Second, there is a change in the age earnings profile associated with the response of the migrant to the prices he faces

[9] By adjusted we mean depreciation corrected. If there was not decpreciation of human capital, i.e. if $\delta_h = 0$, then our measure would be eaxctly that of age earnings profile. Most estimates of δ_h show that it is very small.

in the destination location. The most interesting implications are:

1. The "scale" estimate overpredicts earnings of the migrant at the time of migration. This follows from the fact that $\theta(a_m, a_m) < 0$ and it is driven by the decision on the part of the migrant to engage is more (relative to the stayer) on-the-job training.

2. Over time, the gap between earnings of the migrant and the stayer increase and, at least close to retirement, the differences exceed the differences in wage rates. Thus, the shape of the age earnings profile of migrants and stayers is different, with migrants exhibiting a steeper age-earnings profile.

3. The differences in age-earnings profiles are not independent of the age at migration, a_m. In particular, the older the migrant, the flatter his post-migration age-earnings profile.

Even though human capital differences peak at some pre-retirement age, no such non-monotonicity appears in age earnings profile.

We find these results interesting because they suggest that evidence on the age-earnings profiles of migrants relative to stayers can be used to evaluate whether a model in which investing in human capital is a "rival" activity (e.g. the Ben-Porath model) is more or less consistent with the data relative to the "complementarity" view of on-the-job training (e.g. the learning-by-doing model).

2.1.2. Migrants vs. Natives

It is standard in the literature to compare natives and immigrants with the *same level of schooling* and we pursue the same strategy here, as we spell out the implications of the model for the differences in earnings between those two groups. We concentrate on the case in which migration is, as before, completely unexpected and it takes place after the individual has completed his schooling (in the native location). Moreover, we assume that the relevant price differentials are such that the migrant chooses not to go back to school.[10]

Unlike the previous section the level of human capital of the native and the migrant at the age of migration, $h_n(a_m)$ and $h_m(a_m)$ respectively, are not in general equal. Thus, the differences in age earnings profiles are driven by both the differences in "initial human capital" as well as differences, if any, in the human capital accumulation decisions by the two individuals after age a_m.

In order to make the results in this section comparable to those in the previous one, we assume that the only difference between the two locations is the wage rate and, as before, the wage rate in the receiving location satisfies $(1-\tau^*)w^* = (1+\varepsilon)((1-\tau)w)$, where $(1-\tau)w$ is the after tax wage rate in the sending location. It is convenient to define the (adjusted) difference in income between the two individuals as

$$\Delta_y^n(a,\varepsilon) = e^{\delta_h(a-a_m)}[y_m(a;a_m,w^*,p_w^*) - y_n(a;w^*,p_w^*)],$$

where $y_j(a;w^*,p_w^*)$ is earnings of individual $j \in \{n,m\}$. We summarize the results in the following proposition

Proposition 1. *Let n and m denote variables corresponding to a native and an immigrant to a higher wage location with **the same years of schooling**, and let a_m be the age at which m migrates. Assume that $(1-\alpha_2)\upsilon < \alpha_2$ then:*

1. *At the time of migration, the level of human capital of the native exceeds*

[10] Basically we assume that in the new environment, the optimal solution to the income maximization problem is interior. When ruling out corners we are effectively ruling out not only individuals who transition from being workers in the location of origin to being students in the new location, but also individuals whose first "job" is an unpaid internship.

that of the migrant. Formally, $h_n(a_m) > h_m(a_m)$.

2. *The innate ability of the immigrant is higher than the ability of the native, that is,* $z_{h,m} > z_{h,n}$.

3. *At the time of migration, the income of the immigrant is lower than the income of the native.* $\Delta_y^n(a_m, \varepsilon) < 0$.

4. *The income gap between migrant and natives narrow as a function of time since migration*

$$\frac{\partial \Delta_y^n}{\partial a}(a, \varepsilon) > 0$$

5. *The level of the income gap between immigrants and natives is inversely related to the wage rate (level of development) of the sending location*

$$\frac{\partial \Delta_y^n}{\partial \varepsilon}(a, \varepsilon) < 0$$

6. *The return to experience in the migrants home location is negative; that is*

$$\frac{\partial y_m}{\partial a_m}(a; a_m, w^*, p_w^*) < 0$$

and this gap is increasing in innate ability and the difference in wage rates between the sending and receiving locations.

Proof. [See Appendix] ■

The intuition for the results is as follows. Since the migrant makes his schooling decision in a low wage country he has to have more innate ability (higher z_h) in order to choose to acquire the same number of years of schooling. However, since the "shadow" unit cost of schooling is a combination of the wage rate and market goods, the relative price of market goods is higher in the sending location. This implies that, optimally, fewer market goods are used in producing human capital. Thus, even though both individuals have spent the same amount of time in school, the migrant has done so in schools that do not have as high a quality (as measured by the amount of x_s) as the native and, hence, his human capital at the end of the schooling period is lower. This difference gets bigger if the individual migrates after the completion of schooling. Thus, the initial difference in human capital at the time of migration "explains" the lower wage.

Part two of the proposition shows that the migrant has higher innate ability than the native. Thus, when faced with the same relative prices as the native he rationally chooses to invest more in increasing his human capital (he is more efficient at investing). This has two effects. First, it further contributes to the lower initial earnings as the migrant chooses more "investment friendly" employment options. Second, it implies that the higher human capital accumulated by the migrant results in a steeper age-earnings profile and, hence, in the narrowing of the gap.

Finally, the model predicts that the amount of human capital at the age of migration, $h_m(a_m)$, depends positively on the wage rate in the sending location. Thus, larger wage (or TFP) gaps result in larger differences in earnings upon migration. Moreover, experience in the sending location has a negative impact on the level of income. In section XXXX we describe evidence that is consistent with the implications of the model.

Before we continue, it is relatively straightforward to relax the assumption of equal prices for the inputs in the production of human capital in the three stages: early childhood, schooling and on-the-job training. A brief summary of the effects is as follows (throughout we keep the wage differentials constant):

1. The higher the price of early childhood human capital, p_E, in the sending

location the higher the gap in human capital levels at the time of migration. Higher p_E also imply that the gap in innate skills, z_h, is even larger.

2. Higher levels of p_w have no impact on $h_m(6+s)$ and, hence, no impact on initial earnings differential if migration occurs just after the end of schooling. However, if $a_m > 6+s$ then higher p_w imply larger human capital gaps

3. The unit price of schooling resources, p_s, has no effect. Even though this may seem paradoxical, it is the result of our decision to compare individuals with the same schooling level and this eliminates the impact of p_s. Of course, given a location and a level of innate skill increases in the price of educational inputs result in lower levels of schooling.

Finally, it is interesting to highlight the role played by the presence of early childhood human capital: If all individuals had exactly the same human capital at age 6 then, conditional on choosing to attain a given level of schooling, they all would have exactly the same level of human capital at age $6+s$. Thus, even though the qualitative predictions of the model would be the same, we suspect that the model without early childhood human capital will have difficulty matching the evidence.[11]

[11] At this point we find the evidence on the role of early childhood somewhat difficult to evaluate. On the one hand, Heckman and co-authors (reference needed) claim that most of the variance in human capital is explained by variance in early childhood capital. At the other end, Schoellman (2012) finds that the educational attainment of refugees that arrive in the U.S. does not depend on whether the arrival date is right after birth (which implies that they acquire "U.S. early childhood human capital," or at age five which means that they have "foreign" (and presumably lower quality) early childhood human capital.

2.1.3. The Migration Decision

In the migration literature one of the most studied questions relates to the nature of migration and, in particular, whether migrants are positively or negatively selected (see Borjas (1987) and Grogger and Hanson (2011) for somewhat opposing views). In order to define selectivity we study migration from the "w" country to the "w^*" country. If the individual decides to migrate he incurs two types of costs: He loses a fraction η of his human capital, and he has to pay a fixed cost C. It is straightforward to assume that the fixed cost depends on the wage rates of both the home and the foreign country. The key assumption is that it is not age dependent.[12] For now we ignore taxes, as it is simple to bring them into the model but make notation a little more cumbersome.

A standard argument shows that the present discounted value of income of an individual of age a who is out of school, has human capital h, and lives in the "w^*" location is

$$V(h, a; z_h, w^*) = w^* V_0(a) h + (w^*)^{\frac{1-\gamma_1}{1-\gamma}} V_1(a; z_h),$$

where the function $V(h, a; z_h, w^*)$ gives the present discounted value of income of an a year old individual who has human capital h, who lives in a country with wage w^*, and who has skill level z_h.

In the Appendix we show that

$$V_0(a) = \frac{m(a)}{r + \delta_h},$$

[12] Note that, implicitly, there is age dependency in our cost fucntion as the human capital cost, $\eta h(a)$, depends on the individual's age.

and

$$
\begin{aligned}
V_1(a; z_h) &= \frac{1-\gamma}{\gamma_1}\left[\frac{\gamma_1}{r+\delta_h} z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}\right]^{1/(1-\gamma)} \int_a^R e^{-r(t-a)} m(t)^{1/(1-\gamma)} dt, \\
&= (z_h)^{1/(1-\gamma)} \bar{V}_1 \int_a^R e^{-r(t-a)} m(t)^{1/(1-\gamma)} dt.
\end{aligned}
$$

We assume that when the option to migrate materializes, the payoffs from migrating and staying also include a random component that captures the monetary equivalent of other considerations associated with migrating. Moreover, assume that these random components —independent across individuals— have an extreme value distribution and that their difference, which we denote ε, has a logistic distribution. The individual will choose to migrate if

$$V((1-\eta)h, a; z_h, w^*) - C + \varepsilon \geq V(h, a; z_h, w).$$

To simplify notation, let

$$\Lambda(h, a; z_h, w^*, w) = V((1-\eta)h, a; z_h, w^*) - C - V(h, a; z_h, w),$$

then the fraction of age a individuals with innate ability z_h who chose to migrate is given by

$$P[\varepsilon \geq -\Lambda(h, a; z_h, w^*, w)]$$

Let $N^m(a, z_h)$ be the number of individuals with innate ability z_h who migrate from "w" to "w^*" at age a, and let $N^s(a, z_h)$ be the number of stayers with the same ability level. Given the distributional assumptions, it follows that

$$\ln\left(\frac{N^m(a, z_h)}{N^s(a, z_h)}\right) = \Lambda(h, a; z_h, w^*, w).$$

Since the function $\Lambda(h, a; z_h, w^*, w)$ is increasing in w^* the model predicts that there will be positive sorting: Countries with a higher return to human capita (higher w^* in this model) will attract more immigrants.

Consider now two types of individuals indexed by their innate ability levels, $z_h > z_h'$. It follows (see Manuelli and Seshadri (2013) for details) that $h(a; z_h) > h(a; z_h')$. We refer to the z_h individual as the high skill person, and the z_h' as the low skill worker.

The migration literature defines positive selection as a situation in which the skill composition of the migrants is more tilted towards the high skilled than that of the stayers. Formally, migrants are positively selected if

$$\ln\left(\frac{N^m(a, z_h)}{N^m(a, z_h')}\right) > \ln\left(\frac{N^s(a, z_h)}{N^s(a, z_h')}\right)$$

Since this condition is equivalent to

$$\ln\left(\frac{N^m(a, z_h)}{N^s(a, z_h)}\right) > \ln\left(\frac{N^m(a, z_h')}{N^s(a, z_h')}\right)$$

it is satisfied if and only if

$$\Lambda(h, a; z_h, w^*, w) > \Lambda(h, a; z_h', w^*, w)$$

or, equivalently,

$$(w^*(1-\eta) - w)\frac{m(a)}{r+\delta_h}[h(a; z_h) - h(a; z_h')] + \tag{2.6}$$
$$((w^*)^{\frac{1-\gamma_1}{1-\gamma}} - (w)^{\frac{1-\gamma_1}{1-\gamma}})\bar{V}_1 \int_a^R e^{-r(t-a)} m(t)^{1/(1-\gamma)} dt[(z_h)^{1/(1-\gamma)} - (z_h')^{1/(1-\gamma)}] > 0.$$

There are two cases to consider:

1. Case I: $(w^*(1-\eta)-w)>0$. In this case the model implies that selection is positive. Thus, when the variable component of the cost of migrating is small (i.e. η is small), selection is positive.

2. Case II: $(w^*(1-\eta)-w)\leq 0$. In this case the result can go either way. There are a couple of interesting observations:

    1. If the two countries have similar productivity levels (and hence $w^* \approx w$), then the first term is negative and the second is, by assumption, arbitrarily small. Thus, in the case of the so-called North-North migration, the model predicts that immigrants are negatively selected.

    2. Since it can be shown that

    $$\lim_{a\to R}\frac{\int_a^R e^{-r(t-a)}m(t)^{1/(1-\gamma)}dt}{m(a)}=0,$$

    the model implies that older workers are always negatively selected. The intuition is that older workers do not find profitable to make up for the loss of human capital since their working horizon is shorter.

Our results contrast with the static model typically used in the literature as age at migration plays a role. When $w^*(1-\eta)<w$ it is possible that the nature of selection depends on the age of the migrant. To be precise, for some parameter values it is possible that there is an age $\hat{a}$ such that all migrants younger than $\hat{a}$ are positively selected, while migrants older than $\hat{a}$ are negatively selected. This more complex pattern of migration might explain the instability of the findings based in models that ignore human capital accumulation decisions

2.2. The Option to Migrate: Impact on Human Capital Accumulation

In this section we go beyond the "surprise" migration decision that we studied in the previous section and analyze the impact on the human capital accumulation decision in the sending location of having the option to migrate. Note that the existence of this option will influence the human capital accumulation decisions of all individuals that are eligible to migrate whether they choose to migrate or not.

We model the option to migrate as a two parameter process with a given cost. We denote by λ the Poisson arrival rate of a chance to migrate and by $w^* = (1+\epsilon)w$ the wage in the new location. As before, the cost of migration is given by the pair (η, C).

It is convenient to change slightly the notation that we used for the value function in the previous section. Let $V(h, a; w, w^*, \lambda)$ be the present discounted value of income of an a year old individual who has human capital h, who lives in a country with wage w, and who faces the possibility —which arrives with Poisson parameter λ— of migrating to a region in which the wage rate is w^*. Thus, $V(h, a; w^*, w^*, 0)$ is the value of an individual who lives in the w^* region permanently. Thus, in the previous section we showed that

$$V(h, a; w^*, w^*, 0) = w^* V_0(a) h + (w^*)^{\frac{1-\gamma_1}{1-\gamma}} V_1(a; z_h).$$

Since not everyone will have an option to migrate, and since the decision to migrate —conditional on receiving the option— will depend on the individual's age, the model will predict that migrants and stayers are, on average, heterogeneous. Thus, in this section we ignore the "migration shock" that we described in the previous section and, effectively, set $\varepsilon = 0$.

It follows then that the HJB equation for an individual in the sending location satisfies

$$rV(h,a;w,w^*,\lambda) = \max_{n,x}\{wh(1-n) - x$$
$$+\frac{\partial V}{\partial h}(h,a;w,w^*,\lambda)[z_h(n(a)h(a))^{\gamma_1}x(a)^{\gamma_2} - \delta_h h(a)] + \frac{\partial V}{\partial a}(h,a;w,w^*,\lambda)$$
$$+\lambda[\max(\hat{V}((1-\eta)h,a;w^*,w^*,0) - C, V(h,a;w,w,0)) - V(h,a;w,w^*,\lambda)].$$

In this case, the individual will choose to migrate —conditional on receiving an offer— only if

$$V((1-\eta)h,a;w^*,w^*,0) - C \geq V(h,a;w,w,0).$$

Using our characterization of the value function for stayers (notice that both value functions are for stayers; the only difference is the wage rate), this condition is equivalent to

$$h(a)[w^*(1-\eta) - w] \geq \frac{C - V_1(a;z_h)[(w^*)^{\frac{1-\gamma_1}{1-\gamma}} - w^{\frac{1-\gamma_1}{1-\gamma}}]}{V_0(a)}.$$

It turns out that the nature of the migration decision depends crucially, as before, on the sign of $w^*(1-\eta) - w$. Define the function $\Phi(a;z_h,w,w^*)$ by

$$\Phi(a;z_h,w,w^*) \equiv \frac{C - V_1(a;z_h)[(w^*)^{\frac{1-\gamma_1}{1-\gamma}} - w^{\frac{1-\gamma_1}{1-\gamma}}]}{V_0(a)[w^*(1-\eta) - w]}.$$

Then, there are two cases:

1. Case I: $[w^*(1-\eta)-w] > 0$. In this case the individual considers the possibility of migrating for as long as his (h,a) variables satisfy $h(a) \geq \Phi(a;z_h,w,w^*)$.

2. Case II: $[w^*(1-\eta) - w] \leq 0$. In this case the individual considers the

possibility of migrating for as long as his (h, a) variables satisfy $h(a) \leq \Phi(a; z_h, w, w^*)$.

3. In general, in either case we use the notation $(h, a) \in \Omega(z_h, w, w^*)$ to denote satisfaction of the relevant migration conditions in the two cases.

With this notation in place we can be more explicit about the solution of the HJB equation for the potential migrant. The relevant condition is

$$
\begin{gathered}
rV(h, a; w, w^*, \lambda) = \max_{n,x}\{wh(1-n) - x \\
+\frac{\partial V}{\partial h}(h, a; w, w^*, \lambda)[z_h(n(a)h(a))^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a)] \\
+\frac{\partial V}{\partial a}(h, a; w, w^*, \lambda) + \\
\lambda[V((1-\eta)h, a; w^*, w^*, 0) - C - V(h, a; w, w^*, \lambda)], \text{ for } (h, a) \in \Omega(z_h, w, \hat{w}),
\end{gathered}
$$

and

$$
\begin{gathered}
rV(h, a; w, w^*, \lambda) = \max_{n,x}\{wh(1-n) - x \\
+\frac{\partial V}{\partial h}(h, a; w, w^*, \lambda)[z_h(n(a)h(a))^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a)] \\
+\frac{\partial V}{\partial a}(h, a; w, w^*, \lambda), \text{ for } (h, a) \notin \Omega(z_h, w, w^*).
\end{gathered}
$$

We conjecture that in the "migration region" the function $V(h, a; w, w^*, \lambda)$ is given by

$$
V(h, a; w, w^*, \lambda) = w\bar{V}_0(a; w, w^*, \lambda)h + w^{\frac{1-\gamma_1}{1-\gamma}}\bar{V}_1(a; w, w^*, \lambda) - \frac{C}{r+\lambda},
$$

for some functions $(\bar{V}_0(a; w, w^*, \lambda), \bar{V}_1(a; w, w^*, \lambda))$. Imposing this conjecture on the HJB equation we obtain that

$$
\begin{aligned}
\bar{V}_0(a; w, w^*, \lambda) &= \frac{(w^*/w)(1-\eta)}{r+\delta_h}\left[e^{-(r+\delta_h+\lambda)(R-a)} - e^{-(r+\delta_h)(R-a)}\right] + \\
&\quad \frac{1}{r+\delta_h+\lambda}\left(1+\frac{\lambda(w^*/w)(1-\eta)}{r+\delta_h}\right)\left(1-e^{-(r+\delta_h+\lambda)(R-a)}\right),
\end{aligned}
$$

and

$$
\bar{V}_1(a; w, w^*, \lambda) = \int_a^R e^{-(r+\lambda)(t-a)} M(t; \epsilon, \lambda)dt,
$$

where

$$
M(t; \epsilon, \lambda) \equiv \frac{1-\gamma}{\gamma_1}\left[\frac{\gamma_1}{r+\delta_h} z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}\right]^{1/(1-\gamma)} \bar{V}_0(a; w, w^*, \lambda)^{1/(1-\gamma)}
$$

$$
+\lambda(w^*/w)^{\frac{1-\gamma_1}{1-\gamma}}[\frac{1-\gamma}{\gamma_1}\left[\frac{\gamma_1}{r+\delta_h} z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}\right]^{1/(1-\gamma)} \int_a^R e^{-r(t-a)} m(t)^{1/(1-\gamma)}dt].
$$

The functions $(\bar{V}_0(a; w, w^*, \lambda), \bar{V}_1(a; w, w^*, \lambda))$ have the following properties

Proposition 2. *The functions* $\bar{V}_i(a; w, w^*, \lambda)$, $i = 0.1$ *satisfy*

$$
\frac{\partial \bar{V}_i}{\partial (w^*/w)}(a; w, w^*, \lambda) > 0, \frac{\partial \bar{V}_0}{\partial \lambda}(a; w, w^*, \lambda) > 0, \frac{\partial \bar{V}_i}{\partial a}(a; w, w^*, \lambda) < 0.
$$

A sufficient condition for

$$
\frac{\partial \bar{V}_1}{\partial \lambda}(a; w, w^*, \lambda) > 0,
$$

is that $w^*(1-\eta) - w > 0$. *If this condition does not hold and for* a *close to* R, *the sign is reversed.*

Thus, the marginal return to human capital —the term $\bar{V}_0(a; w, w^*, \lambda)$— as well as the return to on the job training —the term $\bar{V}_1(a; w, w^*, \lambda)$— increase with the wage in the receiving region relative to the home region, and with the (inverse of) the expected time until migration (except as noted).

The stock of human capital accumulated on the job given the chance to migrate is

$$\begin{aligned} h(a; w, w^*, \lambda) &= e^{-\delta_h(a-6-s)} h(6+s; w, w^*, \lambda) + w^{\frac{\gamma_2}{1-\gamma}} [z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}]^{\frac{1}{1-\gamma}} \gamma_1^{\frac{\gamma}{1-\gamma}} \\ & \int_{6+s}^{a} e^{-\delta_h(a-t)} \bar{V}_0(a; w, w^*, \lambda)^{\frac{\gamma}{1-\gamma}} dt, \qquad a \geq 6+s. \end{aligned}$$

Thus, in addition to the effect on $(s, h(6+s; w, w^*, \lambda))$, the schooling variables, the chance to migrate has an impact on the amount of human capital accumulated on the job. The effect is summarized on the impact of the possibility of migration on the term $\bar{V}_0(a; w, w^*, \lambda)$.

We next describe the impact on the schooling variables. We assume that the individual knows that there is a chance to migrate but that this will not materialize until after he is out of school. Before we look at the relevant equations, we need to describe how the level of human capital and the time allocated to human capital accumulation depend on the option to migrate. From the solution to the income maximization problem it follows that

$$n(a)h(a) = w^{\frac{\gamma_2}{1-\gamma}} [z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}]^{\frac{1}{1-\gamma}} \gamma_1^{\frac{\gamma}{1-\gamma}} \bar{V}_0(a; w, w^*, \lambda)^{\frac{1}{1-\gamma}}.$$

Hence, at the end of the schooling period which we identify as the age $6+s$ such that $n(6+s) = 1$, the stock of human capital is

$$h(6+s) = w^{\frac{\gamma_2}{1-\gamma}} [z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}]^{\frac{1}{1-\gamma}} \gamma_1^{\frac{\gamma}{1-\gamma}} \bar{V}_0(6+s; w, w^*, \lambda)^{\frac{1}{1-\gamma}}. \tag{2.7}$$

It can be shown (see Appendix) that human capital at age a given that the individual is in school is given by

$$h(a) = h_E e^{-\delta(a-6)}[1 + B(e^{\mu(a-6)} - 1)]^{\frac{1}{1-\gamma_1}}, \tag{2.8}$$

where

$$\mu = \frac{\gamma_2 r + (1-\gamma_1)\delta_h}{1-\gamma_2},$$

and B is

$$B = \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + (1-\gamma_1)\delta_h} z_h^{\frac{1}{1-\gamma_2}} [\frac{\gamma_2}{p_s} q_E]^{\frac{\gamma_2}{1-\gamma_2}} h_E^{\frac{1-\gamma}{1-\gamma_1}}.$$

Thus, equating equations (2.7) and (2.8) it follows that

$$w^{\frac{\gamma_2}{1-\gamma}}[z_h \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}]^{\frac{1}{1-\gamma}} \gamma_1^{\frac{\gamma}{1-\gamma}} \bar{V}_0(6+s; w, w^*, \lambda)^{\frac{1}{1-\gamma}} = h_E e^{-\delta(s)}[1 + B(e^{\mu(s)} - 1)]^{\frac{1}{1-\gamma_1}}. \tag{2.9}$$

Next, from the solution to the schooling problem (omitted here) we have that

$$w\bar{V}_0(6+s; w, w^*, \lambda) h_E^{\gamma_1} e^{-\gamma_1 \delta s}[1 + B(e^{\mu s} - 1)]^{\frac{\gamma_1}{1-\gamma_1}} = q_E h_E^{\gamma_1} e^{(r+(1-\gamma_1)\delta_h)s} \tag{2.10}$$

Finally, the optimal choice of human capital at age 6, h_E, satisfies

$$h_E = h_B^{\frac{1}{1-\upsilon}} (\upsilon q_E)^{\frac{\upsilon}{1-\upsilon}}. \tag{2.11}$$

Thus, the solution (h_E, q_E, s) to the system of equations (2.9),(2.10) and (2.11) can be used to compute the implications for schooling and for the level of human capital.

Using equation (2.7) we can write

$$h(a;w,w^*,\lambda,z_h) = e^{-\delta_h(a-6-s)}\left[w^{\frac{\gamma_2}{1-\gamma}}[z_h\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}]^{\frac{1}{1-\gamma}}\gamma_1^{\frac{\gamma}{1-\gamma}}\bar{V}_0(6+s;w,w^*,\lambda)^{\frac{1}{1-\gamma}}\right]$$
$$+w^{\frac{\gamma_2}{1-\gamma}}\left[z_h\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}\right]^{\frac{1}{1-\gamma}}\gamma_1^{\frac{\gamma}{1-\gamma}}$$
$$\int_{6+s}^{a} e^{-\delta_h(a-t)}\bar{V}_0(t;w,w^*,\lambda)^{\frac{1}{1-\gamma}}dt, \qquad a \geq 6+s.$$

This condition can be written as

$$h(a;w,w^*,\lambda,z_h) = \left[z_h\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2}\right]^{\frac{1}{1-\gamma}}$$
$$\{e^{-\delta_h(a-6-s)}w^{\frac{\gamma_2}{1-\gamma}}\gamma_1^{\frac{\gamma}{1-\gamma}}\bar{V}_0(6+s;w,w^*,\lambda)^{\frac{1}{1-\gamma}}$$
$$+w^{\frac{\gamma_2}{1-\gamma}}\gamma_1^{\frac{\gamma}{1-\gamma}}\int_{6+s}^{a} e^{-\delta_h(a-t)}\bar{V}_0(t;w,w^*,\lambda)^{\frac{1}{1-\gamma}}dt\}, \qquad a \geq 6+s.$$

Next, describe the migration ocndition (To be completed 4/29/13))

2.3. The Impact of Social Security (incomplete as of 4/29/13)

In this section we study the effect of the generosity and the redistributive nature of the social security regime in the receiving location on the decision to migrate. For simplicity, we ignore the noneconomic components and set $\varepsilon = 0$.

We assume that if net income is denoted $y(a)$ and if an individual has been in the country working from age a_m then his social security wealth is

$$W(a,a_m) = \int_{a_m}^{a} e^{-r(t-a_m)}y(t)dt.$$

Let R denote the retirement age (we assume that everybody retires at R), then upon retirement, the level of benefits is given by

$$b = b_m + b_y W(R, a_m).$$

Then, the present value of income at age R is

$$\int_R^T e^{-r(t-R)}[b_m + b_y W(R, a_m)]dt = \frac{1 - e^{-r(T-R)}}{r}[b_m + b_y W(R, a_m)].$$

Consider first the value function of a non-immigrant who has started contributing to the old age pension regime at age a_m. The relevant HJB equation associated with the income maximization problem is

$$\begin{gathered} rV(h, W, a) = \max_{n,x}\{wh(1-n) - x \\ + \frac{\partial V}{\partial h}(h, W, a)[z_h(n(a)h(a))^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a)] + \frac{\partial V}{\partial a}(h, W, a) \\ + \frac{\partial V}{\partial W}(h, W, a)e^{-r(a-a_m)}[wh(1-n) - x] \end{gathered}$$

$$V(h, W, R) = \frac{1 - e^{-r(T-R)}}{r}[b_m + b_y W(R, a_m)].$$

We conjecture that the function $V(h, W, a)$ has the following form

$$V(h, W, a) = wV_0^R(a; a_m)h + V_1^R(a; a_m, w) + V_2^R(a)W$$

and the appropriate boundary conditions are

$$V_0^R(R; a_m) = 0,\ V_1^R(R; a_m, w) = \frac{1 - e^{-r(T-R)}}{r} b_m,\ V_2^R(R) = \frac{1 - e^{-r(T-R)}}{r} b_y.$$

Define the functions

$$Q(b_y, a_m) = 1 + b_y \frac{1 - e^{-r(T-R)}}{r} e^{-r(R-a_m)},$$

and

$$M^R(t; a_m) \equiv \frac{1-\gamma}{\gamma_1} \left[\gamma_1 z_h \left(\frac{\gamma_2}{\gamma_1} \right)^{\gamma_2} \right]^{1/(1-\gamma)} V_0^R(t)^{1/(1-\gamma)} Q(b_y, a_m)^{-\gamma/(1-\gamma)}.$$

Then, the solution is given by

$$\begin{aligned}
V_0^R(a; a_m) &= Q(b_y, a_m) \frac{m(a)}{r + \delta_h}, \\
V_1^R(a; a_m, w) &= w^{\frac{1-\gamma_1}{1-\gamma}} \int_a^R e^{-r(t-a)} M^R(t; a_m) dt + b_m \frac{1 - e^{-r(T-R)}}{r} e^{-r(R-a)}, \\
V_2^R(a) &= b_y \frac{1 - e^{-r(T-R)}}{r} e^{-r(R-a)}.
\end{aligned}$$

In order to simplify comparisons with the no Social Security case, it is convenient to describe how the new value function relates to the previous one. It follows that

$$\begin{aligned}
V_0^R(a; a_m) &= Q(b_y, a_m) V_0(a), \\
V_1^R(a; a_m, w) &= w^{\frac{1-\gamma_1}{1-\gamma}} Q(b_y, a_m)^{-\frac{\gamma}{1-\gamma}} V_1(a; z_h) + b_m \frac{1 - e^{-r(T-R)}}{r} e^{-r(R-a)}
\end{aligned}$$

We can now describe the relevant HJB equation for a potential migrant. I assume that if migration takes place at age a_m the pension wealth in the recipient country is zero. As before, a hat indicates the recipient country.

$$rV(h, W, a; w, w^*, \lambda, 6+s) = \max_{n,x}\{wh(1-n) - x$$
$$+\frac{\partial V}{\partial h}(h, W, a; w, w^*, \lambda, 6+s))[z_h(n(a)h(a))^{\gamma_1}x(a)^{\gamma_2} - \delta_h h(a)]$$
$$+\frac{\partial V}{\partial a}(h, W, a; w, w^*, \lambda, 6+s) + \frac{\partial V}{\partial W}(h, W, a; w, w^*, \lambda, 6+s)e^{-r(a-6-s)}[wh(1-n) - x]$$
$$\lambda[V((1-\eta)h, 0, a; w^*, w^*, 0, a) - C - V(h, W, a; w, w^*, \lambda, 6+s)],$$
$$\text{for } (h, a, W) \in \Omega^R(z_h, w, w^*),$$

where the set $\Omega^R(z_h, w, w^*)$ is defined as before where the relevant Φ^R function is now

$$\Phi^R(a, W, z_h, w, w^*) = \{C - (\hat{b}_m - b_m)\frac{1-e^{-r(T-R)}}{r}e^{-r(R-a)}$$
$$-V_1(a; z_h)[(w^*)^{\frac{1-\gamma_1}{1-\gamma}}Q(\hat{b}_y, a)^{-\frac{\gamma}{1-\gamma}} - w^{\frac{1-\gamma_1}{1-\gamma}}Q(b_y, 6+s)^{-\frac{\gamma}{1-\gamma}}]$$
$$+V_2^R(a)W\}/\{Q(\hat{b}_y, a)V_0(a)[w^*(1-\eta) - w].$$

The migration rule is as before

1. Case I: $[w^*(1-\eta) - w] > 0$. In this case the individual considers the possibility of migrating for as long as his (h, a, W) variables satisfy $h \geq \Phi^R(a, W, z_h, w, w^*)$.

2. Case II: $[w^*(1-\eta) - w] \leq 0$. In this case the individual considers the possibility of migrating for as long as his (h, a, W) variables satisfy $h \leq \Phi^R(a, W, z_h, w, w^*)$.

3. In general, in either case we use the notation $(h, a, W) \in \Omega^R(z_h, w, w^*)$ to denote satisfaction of the relevant migration conditions in the two cases.

Note that now accumulated pension wealth in the home country is lost if the individual migrates [Should we relax this?] and he starts from zero pension wealth in the recipient country.

[Next step: solve the HJB equation]

3. Confronting the Evidence

In this section we discuss some evidence on the earnings of migrants and use the model to explore if the quantitative predictions match the data.

3.1. Migrants vs. Natives: Evidence

There is a large literature [Note: Cite survey] on the relative income of immigrants and natives. Here we present a sample of the (not always consistent) findings.

Initial Wage Differentials Borjas (1994) estimates that, for recent arrivals, the percentage wage differential between immigrant and native men increased from -16.6% in 1970 to -31.7% in 1990 (see Borjas (1994), Table 3). Borjas (1992) estimates that, in 1970, the average immigrant had .2 years of education less than a native (who had 11.3 years at the time), while in 1990 we estimate that the average immigrant had 12.5 years of schooling (natives had 13).[13] Hanushek and Kimko (2000) also find that immigrants from poorer regions earn less.

More recently, Lemos (2011), using longitudinal data from the U.K. finds that there is a significant gap in initial earnings of immigrants (they earn between 10% and 70% less than natives).

[13] This estimate implies that the gap between immigrants and natives that was estimated to be large in 1980 by Borjas, has narrowed in the 1990s. For evidence on this see Betts and Lofstrom (2000).

Abramitzky et. al. (2011) study assimilation of immigrants to the U.S. early in the 20th Century. They find that migrants from "England, Scotland and Wales held higher paid occupations than U.S. natives upon first arrival ... permanent immigrants from two sending countries that started out with occupation scores below those of natives (Denmark and Portugal) experienced sizable occupation convergence over thirty years." To the extent that England, Scotland and Wales had a level of development that was similar to that of the U.S. around 1900 while Denmark and Portugal's was lower, this is exactly what the selection model implies: catch up by migrants from poor countries, and no differences in earnings in the case of migrants of relatively rich countries.

Borjas (1994) indicates that the elasticity of earnings of migrants with respect to country of origin GDP is around 0.04, while Borjas (2000, Table 1.6 column 4) estimates the elasticity around 0.05. Borjas (1992) reports that the level of GNP per capita in the country of origin of the typical recent immigrant in 1970 was slightly above 50% of the U.S., while in the 1980s (we do not have data for 1990) it had decreased to approximately 39% of U.S. GNP per capita.

Differences in Age-Earnings Profiles Borjas (1994) reports the evidence on the growth rate of earnings of immigrants relative to natives. The precise amount of catch-up is controversial (see Borjas (1994) for a discussion), but it is in the range of 6-15% for the first decade after immigration to 10-25% for the first two decades after immigration.

Lemos (2011) states that in reference to the earnings gap between immigrants and natives "After 10 years, this gap narrows to around zero...After 20 years ...immigrants earn roughly between 0% and 30% more than natives. After 30 years the gap is positive and larger."

The available longitudinal evidence for the U.S. is somewhat mixed. Duleep and Dowhan (2002) find evidence that is consistent with Borjas' results using repeated cross sections, while Kim (2011), using CPS data and, hence, being forced to restrict the analysis to one year, fails to find any evidence of catch up. Lubotsky (2007) using longitudinal data that matches the 1990 and 1991 SIPP data with the 1994 March Supplement to the CPS and earnings shows that selection biases the results from repeated cross sections. In particular, he finds that "immigrant earnings grow by about 10-15 percentage points more over their first 20 years in the United States than the earnings growth experienced by natives." He also finds that misreporting of the arrival rate also biases the measures of the initial earnings gap and he estimates that "the decline in the level of earnings between 1960 arrivals and 1985 arrivals is approximately one-third smaller" when the right date is used.

The Effect of Experience in the Sending Location In a recent study, Goldmann et. al. (2011) study the returns to different form of human capital to immigrants to Canada and find that the "rate of return to pre-immigration labour market experience is negative and statistically significant." Even though as a first pass this seems counterintuitive, it is precisely what the model implies (see Proposition 1).

3.2. Migrants vs. Natives: Model Predictions

As a first pass, we used the parameterized version of the model in Manuelli and Seshadri (2013) to evaluate the performance of the model. To be precise, the model was parameterized to match the U.S. economy (basically ignoring migration) circa 2000. Since there is no consensus about the size the of the differences and the catch-up rate, we decided to use Borjas' since he covers several dimensions and are consistent with each other. We repeat the findings and we add the model's predictions.

Fact 1 Borjas (1994) estimates that, for recent arrivals, the percentage wage differential between immigrant and native men increased from -16.6% in 1970 to -31.7% in 1990 (see Borjas (1994), Table 3). In order to estimate the implications of the model, we need time series estimates of the schooling levels for natives and immigrants, as well as the 'identity' of their country of origin, so that we can estimate the change in 'quality' of the human capital of the average immigrant. Borjas (1992) estimates that, in 1970, the average immigrant had .2 years of education less than a native (who had 11.3 years at the time), while in 1990 we estimate that the average immigrant had 12.5 years of schooling (natives had 13).[14] He also reports that the level of GNP per capita in the country of origin of the typical recent immigrant in 1970 was slightly above 50% of the U.S., while in the 1980s (we do not have data for 1990) it had decreased to approximately 39% of U.S. GNP per capita. Using those values the model predicts that initial —defined as the average over the first five years after immigration— earnings of the average immigrant are 15% lower than those of the natives in 1970, and 23% in 1990. The model is consistent with the view that the 'quality' of the average immigrant has decreased, and this is one reason why recent immigrants earn less than natives (see Borjas (1994)). [15]

[14] This estimate implies that the gap between immigrants and natives that was estimated to be large in 1980 by Borjas, has narrowed in the 1990s. For evidence on this see Betts and Lofstrom (2000).

[15] The model underpredicts the drop in income. However, this is due to our choice of concentrating on selecting immigrants in terms of z_h. If immigrants were selected only in terms of diffferences in h_B, the model predicts differences of -37% and -51%. Thus if the proportion of 'z_h immigrants' was 92% in 1970 and 68% in 1990, the model would perfectly predict the observed differences in income. The increase in the proportion of immigrants who gain less from immigrating is consistent with the change in U.S. immigration policy that reduced the number of 'economic' migrants in favor of individuals with family ties.

Fact 2 Borjas (1994) reports the evidence on the growth rate of earnings of immigrants relative to natives. The precise amount of catch-up is controversial (see Borjas (1994) for a discussion), but it is in the range of 6-15% for the first decade after immigration to 10-25% for the first two decades after immigration. We analyzed the 10 and 20 year average growth rate of earnings (relative to natives with the same years of schooling) for two individuals: one that comes from a country in the middle of the world income distribution and the other that comes from a country in the lowest decile. As before, we considered both individuals that differ in terms of their h_B, as well as immigrants who differ (from their fellow country men) in terms of z_h.[16] The results (the first number corresponds to h_B, while the second gives the predictions for z_h) are presented in Table 7. Our estimates fall within the range reported in the literature and capture the actual amount of catch-up

Growth Rate of Relative Income		
GNP (origin)	10-year	20-year
Middle Income	5-14%	7-20%
Low Income	8-19%	11-27%

Fact 3 We used the model to estimate the level of initial income (first five years) for an individual with 12 years of schooling as a function of output per worker in his country of origin. We computed earnings in the U.S. of an immigrant from a middle income country (50% of U.S. output per worker) and a poor country (7% of U.S. output per worker). Given the non-linearity of the model, the computed elasticity is sensitive to the choice of country

[16] As before, when we "endow" an individual with a different level of z_h (or h_B) we solve the "new" income maximization problem faced by that individual, and we use the values of human capital corresponding to the new solution.

(as well as source of variation). We find estimates that range from 0.01 to 0.04 (when only z_h is varied) to 0.05 to 0.17 (when only h_B is varied). Borjas (1994) indicates that the elasticity of earnings with respect to GDP is around 0.04, while Borjas (2000, Table 1.6 column 4) estimates the elasticity around 0.05. Thus, the model's predictions are roughly in agreement with the evidence.

3.2.1. Accounting for Hendricks' Evidence

In research that is quite close in spirit to our work, Hendricks (2002), uses earnings of immigrants to the U.S. to estimate the human capital in the country of origin.[17] He concludes that human capital differences cannot account for a large share of the cross-country variation in output per worker. In Hendricks' framework — and ignoring self-selection— if the 'efficiency' of human capital in country j is close to one, then the Mincer estimate of human capital is approximately correct, and the approach pioneered by Klenow and Rodriguez-Clare (1997), and Hall and Jones (1999) yields the correct estimate of human capital for each country. Hendricks' estimates of the efficiency parameter are quite high, with only 7 out of 67 countries in the sample displaying efficiency levels below 80% of the U.S., and the 'low income' sample having an average efficiency level of 90% of the U.S.[18]

Hendricks' results are not inconsistent with the model in this paper, even

[17] See also Schoellman (2010) who estimates the returns to schooling in the U.S. and finds that returns to schooling increase with the level of development of the country of origin. It is not strightforward to interpret the results in light of the model in this paper since schooling is endogenous and, hence, if schooling varies across individuals there are a number of factors that must vary as well. The implications of the model along these lines are left for future work.

[18] The efficiency levels in the one type of human capital model can be estimated from the information in Table 1 (p. 204) in Hendricks (2002). They are given by the ratio w_c^η / w_c^N, which can be inferred from the Table. Hendricks sample is not comparable to ours. There is no data for any country in the lowest decile, and data for only two countries for the second poorest decile. We thank Lutz Hendricks for his help in interpreting his estimates.

though his interpretation is. In order to properly evaluate the results it is necessary to use a model that explains the determinants of schooling. To be precise, it is necessary to take a stand on the reasons why an immigrant to the U.S. from the second lowest decile in Hendricks' sample[19] has more than 15 years of schooling, while the average years of a worker schooling that did not migrate is 4.64. One possibility is to assume that observed years of education and ability are not correlated. Thus, the ability of the migrant is similar to the ability of a native (U.S.) worker. In this view, differences in schooling must be due to shocks and Hendricks' conclusion is correct. In the framework of this paper more able individuals acquire, in equilibrium, more schooling. Thus, in our view an individual who has acquired 15 years of schooling when the average in his country of origin is just 5, has a higher ability level. To check if the implied (by the model) differences in ability is consistent with the evidence presented by Hendricks we followed the same strategy we used before: we picked a z_h (for the future migrant to the U.S.) to match the level of schooling of immigrants.[20] In this case, our estimates of efficiency are around 80-90%.[21]

We conclude that Hendricks' evidence is not inconsistent with our model, although his interpretation of it is. However, we conclude that our view is a more reasonable description of the evidence on immigrant earnings. The reason for this is simple: Under Hendricks' interpretation, natives and immigrants with the same level of schooling should behave in similar ways. The evidence surveyed by Bor-

[19]These countries are Ghana and Kenya. Ethiopia, which is in the poorest 10%, is listed in the Appendix, but no data are presented in Table 1.

[20]This implies that this "high z_h" indvidual invests more in human capital than the "average z_h" individual in his country of origin. Thus, when he "arrives" in the U.S. (with 15 years of schooling), his human capital is significantly higher than the average for his country of origin.

[21]Hendricks finds that for many countries, including some relatively poor countries, the efficiency parameter exceeds 100%. Our model, by construction, has to find efficiency levels below 100%. Of course, this ignores selection in other dimensions.

jas (1994, 2000) clearly shows that this is not the case. On the other hand, our approach is consistent with both the catch up data and the change of origin evidence, which suggest that selection —our main driving force— plays a substantial role in explaining immigrant earnings.

The Table presented below demonstrates what the model predicts about the earnings of immigrant relative to natives with the same years of schooling. Note that individuals from poorer countries need to be much more able (higher z_h) than their American counterparts in order to rationalize the same schooling choice.

Immigrant Data

y	s	s	Earnings (rel. to U.S.)		z_h
	B-L	immigrant	Data (%)	Model (%)	U.S. = 0.36
0.89	10.0	14.4	127.9	97.7	.387
0.81	9.26	13.9	125.7	96.1	.392
0.71	9.44	13.4	117.5	94.9	.396
0.63	9.2	14.1	115.9	93.3	.401
0.51	8.1	13.0	97.9	92.6	.403
0.44	6.8	11.8	92.4	90.1	.406
0.33	6.2	13.0	91.9	87.2	.411
0.25	6.5	12.9	99.8	82.7	.417
0.16	5.0	12.5	82.6	78.1	.419
0.06	3.8	13.8	82.7	70.2	.423

Note that all of the relative earnings numbers are below a 100. This is because in the above Table we assumed that the only source of variability is z_h. By contrast if some of the variability was caused by differences in h_B, then these numbers will be above 100.

3.3. Migrants vs. Stayers: Evidence

Jasso and Rosenzweig (2002) analyze earnings of immigrants in their country of origin and in the U.S. They report differences in wages that correspond to average 'before' and 'after' migration wages. Large differences are interpreted as evidence of substantial TFP differences. Their findings suggest that, on average, migration is associated with significant increases in earnings. The average value masks a substantial amount of heterogeneity. For example, Jasso and Rosenzweig report that 24% of the sample earned less in the U.S. than in their country of origin. The small sample size (230 observations), the imputation criteria[22], and the lack of data that would allow us to use our model to predict earnings[23], makes us cautious about using these estimates.

An additional piece of evidence is provided by Chiquiar and Hanson (2005). They find that the average wage of a high school graduate who migrated to the United States relative to the average wage of a high school graduate who chose to stay back in Mexico at age 30 is around 1.43.

3.4. Migrants vs. Stayers: Model Predictions

We used the model —as before, we use the parameters from Manuelli and Seshadri (2013)— to estimate the predictions about the age-earnings profile of a Mexican immigrant like the ones studied by Chiquiar and Hanson (2005).

In 2000, Mexican GDP per worker was around 40% of that of the US. Years of schooling in Mexico were around 6 while that in the US was around 12. Since the average Mexican migrates at age 21, it is safe to assume that the average

[22] Jasso and Rosenzweig imputed full time earnings on the basis of their information. They do not report what fraction of the sample had its earnings inputed.

[23] To be precise, we need data on the time elapsed between last job in the country and first job in the U.S. so that we can use our model to adjust for changes in human capital.

migrant acquired most of his schooling in Mexico. Now we turn to examining the predictions of our model. Imagine that we were to choose the level of TFP that matched the relative GDP per worker differentials. It follows that

$$\frac{w^{US}}{w^{MEX}} = \left(\frac{z^{US}}{z^{MEX}}\right)^{1/(1-\theta)} = (1.136)^{1/(1-0.315)} = 1.2.$$

Notice that even though TFP in the US is only about 13.6% higher than Mexico, the average wage per effective unit of human capital is amplified by the capital's share of income, θ. Next, we need to compute the stock of human capital of a high school graduate in Mexico. Since the average Mexican goes to school for only about 6 years, something needs to change at the micro level in order for the hypothetical individual to endogenously acquire 12 years of schooling. Assume that the ability level, z_h is adjusted upwards so as to induce the individual to acquire 12 years of schooling. We consider two cases, one wherein migration unexpectedly happens at age 21 and the other extreme wherein at birth, the individual knew that he was going to migrate to the United States at age 21.

Case 1: Migration Unexpected When migration unexpectedly happens, the migrant to the United States faces a higher effective (after tax) wage rate that is about 20% higher. Furthermore, since he now faces a higher wage rate, he engages in more on the job training. Consequently, we need to re-calculate his optimal profile for human capital investment. The resulting wage ratio turns out to be given by

$$\left(\frac{w_{Mig}^{HS}}{w_{\mathrm{Re}\,s}^{HS}}\right)_{Age=30} = 1.34.$$

Case 2: Migration Expected Suppose instead that migration is decided upon at birth. In other words, since the individual knows with certainty that he will migrate at age 21, the relevant wage rate is, for all practical purposes, the US wage rate. Again as in the previous case, imagine that z_h is adjusted so as to obtain 12 years of schooling as the optimal choice. (Since the decision problem is almost identical to that of the average American who goes to school for 12 years, z_h needs to change only slightly) When migration happens at age 21, the individual does not alter much his human capital investment profile since he had anticipated this event. Consequently, his human capital profile looks very similar to the average US high school graduate. The resulting wage ratio then reads

$$\left(\frac{w_{Mig}^{HS}}{w_{\mathrm{Re}\,s}^{HS}}\right)_{Age=30} = 1.93.$$

The two extremes give us bounds between which the data should lie and rather remarkably, the data does indeed lie in between. The fact that the data point lies closer to our lower bound suggests that individuals expect migration to take place at an age closer to age 21 than age 0.[24]

Overall, we conclude that the evidence on immigrant income lends support to the view that at least some of the differences in output per worker are driven by differences in the quality of human capital.[25]

[24] If the potential Mexican inmigrant "learns" at age 18 that he will migrate to the U.S. at age 21 (and therefore starts adjusting his human capital correspondingly), the model predicts the wage differential at age 30 exactly at 43%.

[25] In our discussion we completely ignored the impact of differences in languages and learning about the host country environment. These are important considerations and a search model or a set-up along the lines of Jovanovic (1979) can also account for steeper age-earnings profiles and lower initial wages. These generalizations are beyond the scope of this paper. For a nice exposition of other theories of of the earnings distribution, see Neal and Rosen (1999).

3.5. The Impact of the Option to Migrate: Evidence

The model implies that the possibility of migration to a higher income country should raise human capital accumulation for all eligible individuals in the home country. Shrestha (2011) describes the results of a policy change by the British army that comes close to replicating the conditions of the model. In 1993 the British army increased the educational requirement for individuals that wanted to join the Gurkha regiments. Using a difference in difference estimator (the control group was the non-Gurkha populations), Shresta shows that the policy change resulted in an increase of 1.2 years of education over and above what is explained by other factors by the small chance of joining the British army. Since at the time of the implementation of the program, average years of education were less than six for the eligible population, the small chance of "migrating" (the British army recruits only 300 individuals per year) had a relatively large impact on the educational achievement of the eligible ethnic groups.

Chand and Clemens (2008) study the impact of the deterioration of internal economic conditions for Fijians of Indian origin as a result of a series of coups by native Fijian army officers starting in 1987. At the same time that economic conditions in the home country got worse the possibility of migration was tied to investments in schooling since the most likely destination countries (Australia, New Zealand and Canada) all adopted a point system that rewarded potential migrants based on their educational achievements. These developments correspond to an increase in the ratio w^*/w and an increase in λ. Chand and Clemens find that the Indian population that did not migrate increased their educational attainment relative to the native Fijian population, and that the increase was mostly concentrated at the tertiary level

3.6. The Effect of the Welfare State: Evidence

Razin and Wahba (2011) develop a model of the impact of migration in a "free" (or unrestricted) migration regime and show that more generous welfare payments (and they include old age pensions in the empirical measure) result that immigrants are negatively selected. They also consider a political equilibrium in which domestic voters choose the skill composition of the migrants. They show that if the decisive voter is high skill an exogenous increase in taxes will induce him to "vote" for more skilled immigrants as a way of sharing the burden of a more generous welfare state. Razin and Wahba looked at the European Union plus Norway and Switzerland as the recipient countries. They view immigration from other EU countries as driven by the unrestricted regime, while immigration from other developed (e.g. USA, Hong Kong) or developing (e.g. Argentina, India, Nigeria) countries is viewed as driven by the restricted regime.

They use the difference between the high skilled stock of migrants divided by the population (same skill) in the sending country and the same measure for low skill as their measure of composition (Note: Why is it necessary to deflate by the population of the sending country?). They find that in the free regime higher benefits are associated with negative selection, while in the restricted regime the coefficient has the opposite sign. They conclude that in the case of free or unrestricted migration more generous welfare payments induce negative selection, while in the case of restricted migration more generous welfare benefits are associated with positive selection.

There are a number of econometric issues that need to be looked at very carefully.

4. Conclusion (and "to do" list)

The human capital accumulation model of Ben-Porath appears to be broadly consistent with the evidence on the dynamics of immigrant earnings. The next step will probably require understanding how the costs of migration vary across occupations and ages to estimate the demand for migration spots. Additionally, it seems that the model is well suited to analyzing the impact upon human capital formation of an "aid strategy" based on a migration lottery.

5. Appendix

Migrants vs. Stayers It is possible to show that earnings at age a, given that human capital at age a_m is $h(a_m)$ is given by

$$y(a,w,p_w) = (1-\tau)we^{-\delta_h(a-a_m)}\{h(a_m)$$
$$+\frac{r+\delta_h}{\gamma_1}\left(\frac{\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} \int_{a_m}^{a} e^{-\delta_h(a_m-t)} m(t)^{\frac{\gamma}{1-\gamma}} dt$$
$$-\frac{\gamma}{\gamma_1}\left(\frac{\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} e^{\delta_h(a-a_m)} m(a)^{1/(1-\gamma)}\}.$$

Thus,

$$y(a,w,p_w) = (1-\tau)we^{-\delta_h(a-a_m)}\{h(a_m)+$$
$$\left(\frac{\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} \frac{1}{\gamma_1}\times$$
$$\left[(r+\delta_h)\int_{a_m}^{a} e^{-\delta_h(a_m-t)} m(t)^{\frac{\gamma}{1-\gamma}} dt - \gamma e^{\delta_h(a-a_m)} m(a)^{1/(1-\gamma)}\right]\}.$$

Let $\theta(a,a_m)$ be given by

$$\theta(a,a_m) = (r+\delta_h)\int_{a_m}^{a} e^{-\delta_h(a_m-t)} m(t)^{\frac{\gamma}{1-\gamma}} dt - \gamma e^{\delta_h(a-a_m)} m(a)^{1/(1-\gamma)}.$$

which implies that

$$e^{\delta_h(a-a_m)} y(a,w,p_w) = (1-\tau)w\{h(a_m)+$$
$$\left(\frac{\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_2} z_h \left(\frac{(1-\tau)w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} \frac{1}{\gamma_1}\times\theta(a,a_m)\}.$$

Then, simple calculations show that

$$\theta(a_m, a_m) < 0,\ \ \theta(R, a_m) > 0 \text{ and} \frac{\partial \theta}{\partial a}(a, a_m) > 0.$$

Migrants vs Natives In order to show the result, let us assume that the stock of human capital at age 6, h_E, is given (we will endogeneize this later.) Let the costate variable associated with the Hamiltonian in the individual maximization problem be denoted $q(t)$. Then, it follows that the Euler equation during the schooling period (i.e. during the period such that $n(a) = 1$) is simply

$$\frac{\dot{q}(t)}{q(t)} = (r + \delta_h) - [z_h \gamma_1 h(t)^{\gamma_1 - 1} x(t)^{\gamma_2}],$$

while the law of motion for human capital is

$$\frac{\dot{h}(t)}{h(t)} = z_h h(t)^{\gamma_1 - 1} x(t)^{\gamma_2} - \delta_h h(t).$$

It follows that

$$\frac{\dot{q}(t)}{q(t)} + \gamma_1 \frac{\dot{h}(t)}{h(t)} = r + (1 - \gamma_1)\delta_h.$$

It is possible to show that the following conditions must hold

$$q(t)h(t)^{\gamma_1} = q_E h_E^{\gamma_1} e^{(r+(1-\gamma_1)\delta_h)(t-6)}$$

where q_E is the initial level of the costate variable.

Next. Let us look at "optimal" law of motion for the stock of human capital. Given these results, the law of motion of human capital can be written as

$$\dot{h}(t) = z_h h(t)^{\gamma_1} [\frac{\gamma_2 z_h}{p_s} q(t) h(t)^{\gamma_1}]^{\gamma_2/(1-\gamma_2)} - \delta_h h(t), \qquad t \in [6, 6+s).$$

Some additional substitution implies that

$$\dot{h}(t) = z_h h(t)^{\gamma_1} [\frac{\gamma_2 z_h}{p_s} q_E h_E^{\gamma_1} e^{(r+(1-\gamma_1)\delta_h)(t-6)}]^{\gamma_2/(1-\gamma_2)} - \delta_h h(t)$$

or

$$\dot{h}(t) = z_h [\frac{\gamma_2 z_h}{p_s} q_E h_E^{\gamma_1}]^{\gamma_2/(1-\gamma_2)} e^{\frac{\gamma_2}{1-\gamma_2}(r+(1-\gamma_1)\delta_h)(t-6)} h(t)^{\gamma_1} - \delta_h h(t).$$

The solution to this equation must satisfy must satisfy $h(6) = h_E$.

Claim 1. *The solution to the previous equation is given by*

$$h(t) = h_E e^{-\delta(t-6)} [1 + B(e^{\mu(t-6)} - 1)]^{\frac{1}{1-\gamma_1}},$$

where

$$\mu = \frac{\gamma_2 r + (1-\gamma_1)\delta_h}{1-\gamma_2},$$

and B *is given by*

$$B = \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + (1-\gamma_1)\delta_h} z_h^{\frac{1}{1-\gamma_2}} [\frac{(1-\tau)w}{p_w} \frac{q_E}{(1-\tau)w} \gamma_2]^{\frac{\gamma_2}{1-\gamma_2}} h_E^{-\frac{1-\gamma}{1-\gamma_2}}$$

Proof. Simple computation. ■

It is possible to show that since

$$\frac{\dot{q}(t)}{q(t)} = r + (1-\gamma_1)\delta_h - \gamma_1 \frac{\dot{h}(t)}{h(t)}$$

the shadow price of human capital satisfies (for a given s)

$$q(t) = \frac{q_E e^{(r+\delta_h)(t-6)}}{[1 + B(e^{\mu(t-6)} - 1)]^{\frac{\gamma_1}{1-\gamma_1}}},$$

Since it is possible to show that

$$q(6+s) = (1-\tau)w\frac{m(6+s)}{r+\delta_h},$$

it must be the case that, for a given level of schooling

$$\frac{m(6+s)}{r+\delta_h} = \frac{q_E}{(1-\tau)w}\frac{e^{(r+\delta_h)s}}{[1+B(e^{\mu s}-1)]^{\frac{\gamma_1}{1-\gamma_1}}}.$$

Lemma 2. *For a given level of schooling, s, the adjusted shadow price of human capital at age 6, $q_E/((1-\tau)w)$. is independent of any other prices, the level of early childhood human capital, h_E, and the level of innate ability, z_h.*

Proof. [To be added] ■

Lemma 3. *For a given level of schooling, s, the stock of human capital at the end of the schooling period, $h(6+s)$, is proportional to the level of early childhood human capital and independent of price variables and innate ability.*

Proof. [To be added] ■

Lemma 4. *For a given level of schooling, s, the innate ability of a migrant from a low wage location must be lower than the innate ability of a native if:*

1. *If the price of the inputs in the early childhood human capital are the same in both locations, then this result obtains if and only if $(1-\alpha_2)\upsilon < \alpha_2$.*
2. *If the price of the inputs in the early childhood human capital is proportional to the wage rate in each location (labor intensive technology) then migrants have higher innate ability for all parameter configurations*

Proof. [To be added] ■

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